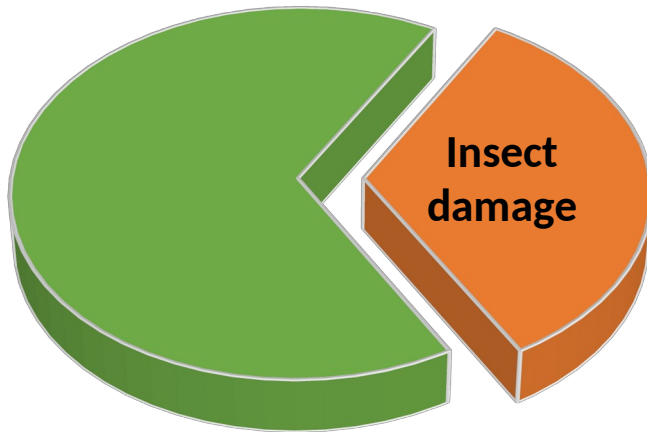


INRAE

➤ Inverse problem for locating insect pests using pheromone detection

Pheromone detection for precision cropping

Insects destroy 1/3 of crop production

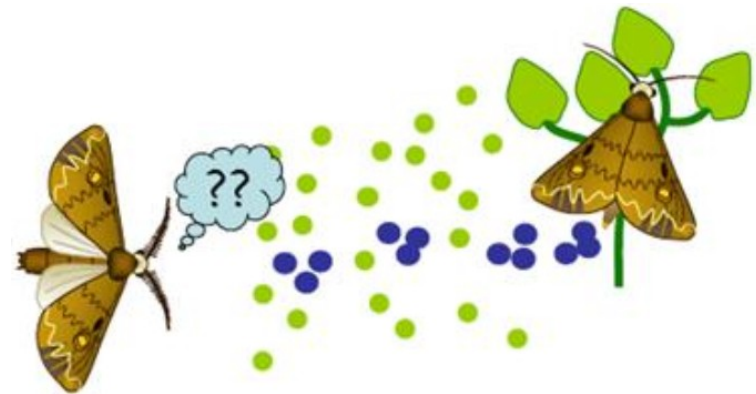


- Insects communicate with species-specific pheromones
- Pheromone system is unique in terms of sensitivity and specificity

Early detection of insect pests
(before infestation settles)

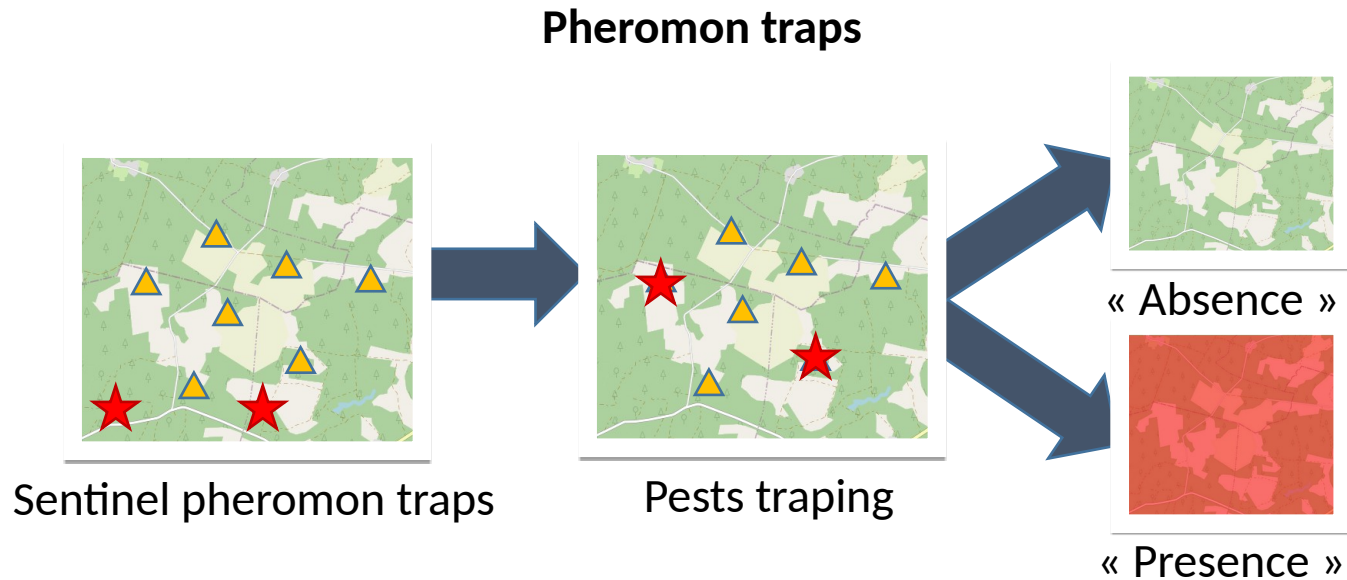


Targeted biocontrol strategies
Incidence of prophylactic approaches



From P.Lucas

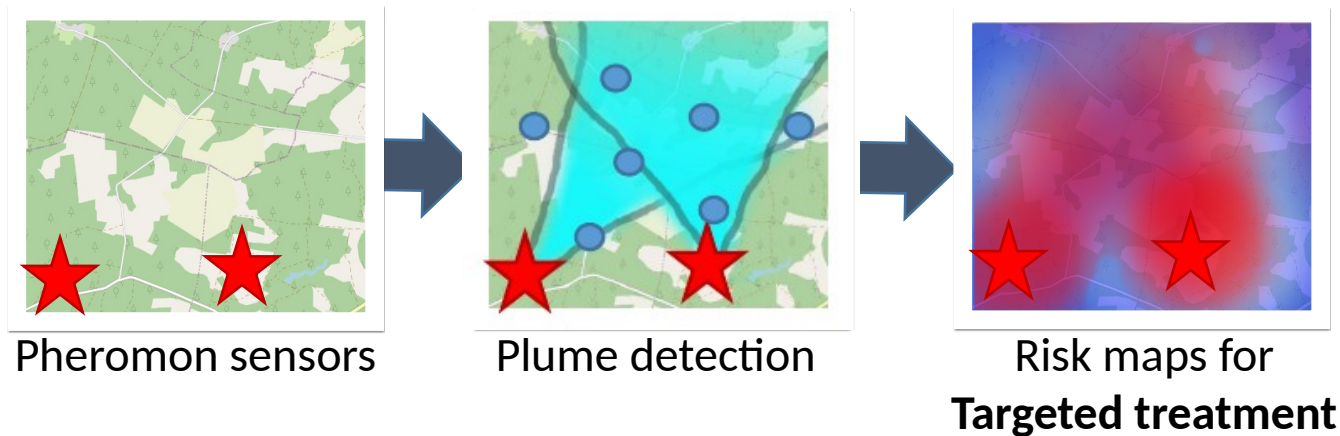
Sensors should provide more precision to build risk maps



Pests natural location
still unknown

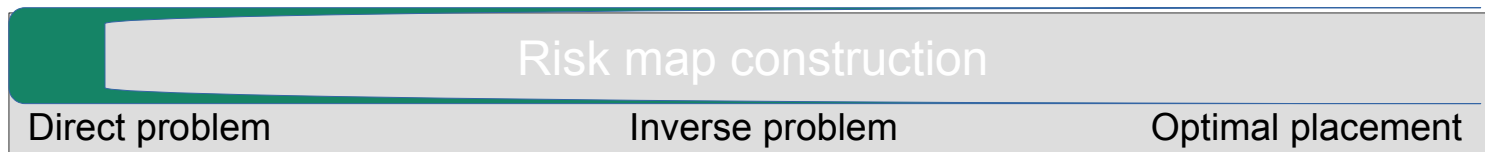
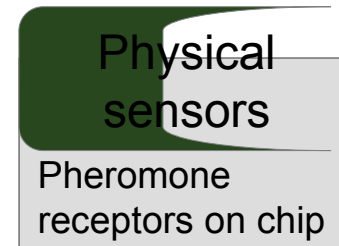
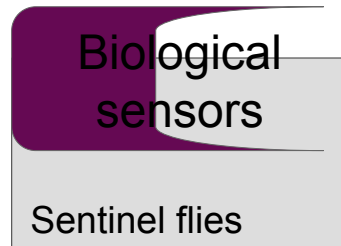
Sensors should provide more precision to build risk maps

Pheromon sensors



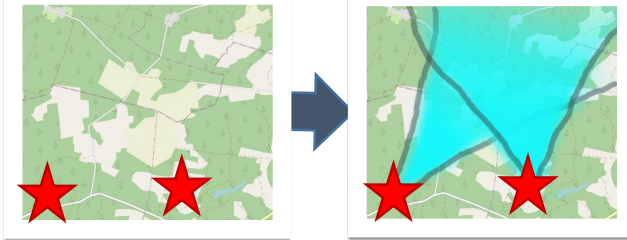
Pests localization for targeted treatment in their natural habitats

Pherosensor



Different mathematical problems

Direct problem



Knowing : pest location + environmental covariables

Predict : pheromone plumes

Inverse problem



Knowing : sensor location and signals + environmental covariables

Predict : pheromone plumes sources (i.e. pests location)

Optimal design

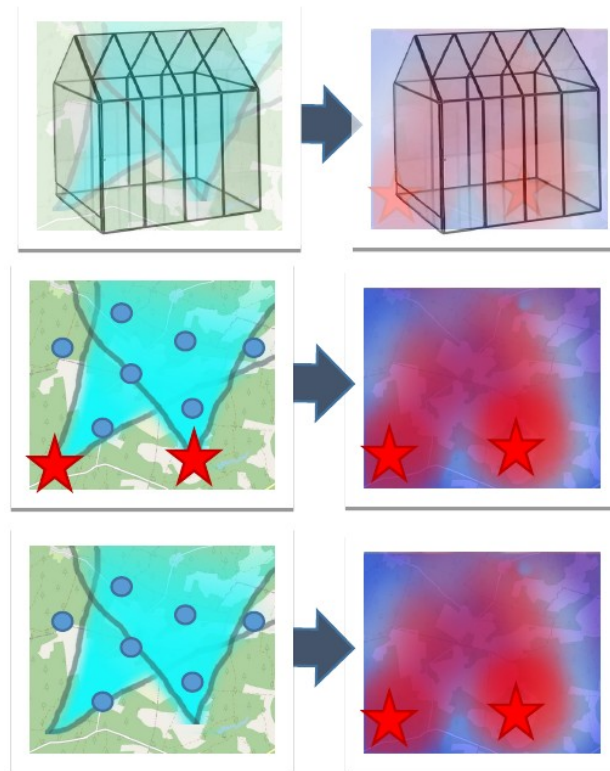


Knowing : environmental covariables

Predict : optimal sensor placement for reduced risk map uncertainty

Risk maps assessment

Risk maps in decreasingly uncontrolled environments



Relaxing experimental control

Modeling odor plume dispersion including environmental conditions

Environmental input data:

- wind field $\vec{u}(x, y, t)$,
- isotropic diffusion tensor $\mathbf{K}(x, y, t)$,
- **vegetation dependent loss coefficient** $\tau_{loss}(x, y)$.

Direct problem

State variable:

concentration in pheromone c .

Task 3.1.1: 2D model for pheromone dispersion:

$$\frac{\partial c}{\partial t} - \underbrace{\nabla \cdot (\mathbf{K} \nabla c)}_{\text{diffusion}} + \underbrace{\nabla \cdot (\vec{u} c)}_{\text{wind}} + \underbrace{\tau_{loss} c}_{\text{deposition}} = \underbrace{s}_{\text{emission}}$$

Quantity of pheromone emitted by pest $s(x, y, t)$.

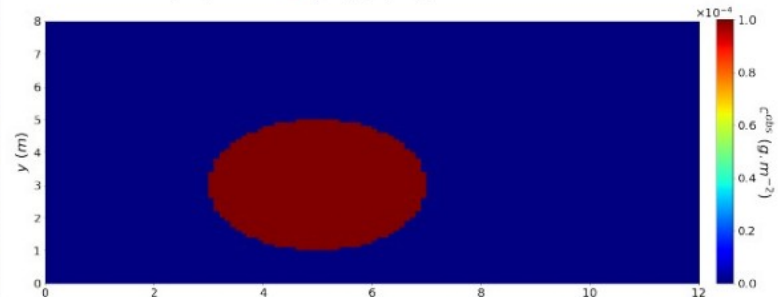


Figure: Pest density p uniformly located in a circle with a constant emission rate q leads to s^{circ} .

Concentration in pheromone c given a source term s .

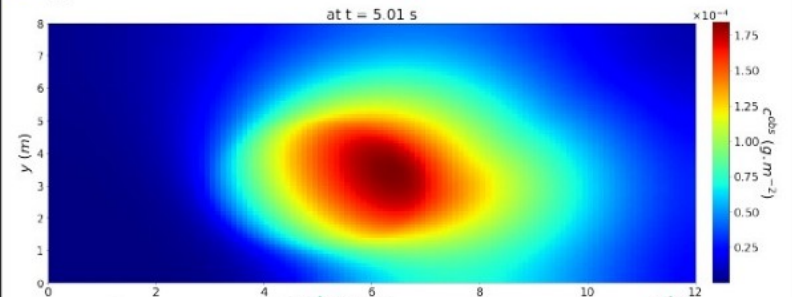


Figure: Concentration $c(s^{circ})$ With the circular source s^{circ} , at a given time.

Inverse problem to build risk maps

Sensors

Observations m^{obs}

Task 3.2.1: Optimization problem:

$$\begin{cases} \text{find } s_a(x, y, t) \text{ such that} \\ s_a(x, y, t) = \arg \min_{s(x, y, t)} (j_{obs}(s) + j_{reg}(s)) \end{cases}$$

with j_{reg} a regularization term and

$$j_{obs}(s) = \|m(c(s)) - m^{obs}\|_2^2.$$

Prediction of the observed variable $m(c(s))$.

Direct model

Sensors

Inverse problem
Control variable: pest density s .

Optimal source term s_a :
pest distribution that makes the direct model fits at best the observations m^{obs} .

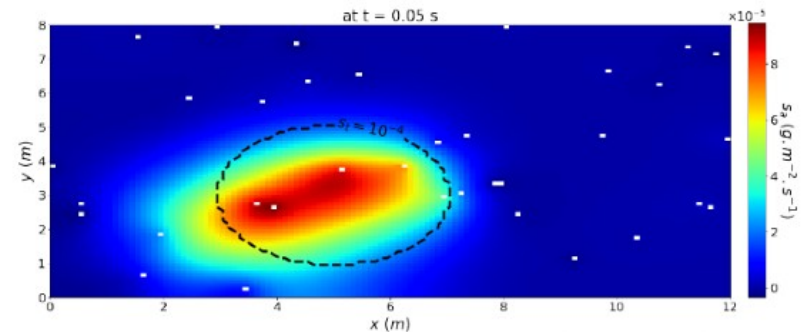


Figure: Optimal source term s_a and target s_t (dashed line). The white dots are the locations of the sensors.

Finite difference vs Variational method

Finite difference

compute $\langle \nabla j_{obs}(s), \delta s \rangle \simeq \frac{j_{obs}(s + \varepsilon \delta s) - j_{obs}(s)}{\varepsilon}$, for all δs

$(N_x \times N_y \times N_t + 1)$ PDE resolution



Finite difference vs Variational method

Variational method

$$\begin{aligned}\langle \nabla j_{obs}(s), \delta s \rangle &= \langle \nabla_m j_{obs}, \partial_c m \cdot \partial_s c \cdot \delta s \rangle \\ &= \langle (\mathbf{f} \partial_c m)^* \nabla_m j_{obs}, w \rangle\end{aligned}$$

with $w = \partial_s c \cdot \delta s$

Finite difference vs Variational method

Variational method

$$\begin{aligned}\langle \nabla j_{obs}(s), \delta s \rangle &= \langle \nabla_m j_{obs}, \partial_c m \cdot \partial_s c \cdot \delta s \rangle \\ &= \langle (\partial_c m)^* \nabla_m j_{obs}, w \rangle\end{aligned}$$

$$\text{with } w = \partial_s c \cdot \delta s$$

Writing the PDE equation as $\mathcal{L}(s, c(s)) = 0$ We have (tangent linear)

$$d_s \mathcal{L}(s, c(s)) \cdot \delta s = \partial_s \mathcal{L}(s, c(s)) \cdot \delta s + \partial_c \mathcal{L}(s, c(s)) \cdot w = 0$$

So that

$$\langle \nabla j_{obs}(s), \delta s \rangle = \langle (\partial_c m)^* \nabla_m j_{obs}, w \rangle + \langle \partial_s \mathcal{L}(s, c(s)) \cdot \delta s, z \rangle + \langle \partial_c \mathcal{L}(s, c(s)) \cdot w, z \rangle$$

for any z



Finite difference vs Variational method

Variational method

$$\begin{aligned}\langle \nabla j_{obs}(s), \delta s \rangle &= \langle \nabla_m j_{obs}, \partial_c m \cdot \partial_s c \cdot \delta s \rangle \\ &= \langle (\partial_c m)^* \nabla_m j_{obs}, w \rangle\end{aligned}$$

$$\text{with } w = \partial_s c \cdot \delta s$$

Writing the PDE equation as $\mathcal{L}(s, c(s)) = 0$ We have (tangent linear)

$$d_s \mathcal{L}(s, c(s)) \cdot \delta s = \partial_s \mathcal{L}(s, c(s)) \cdot \delta s + \partial_c \mathcal{L}(s, c(s)) \cdot w = 0$$

So that

$$\langle \nabla j_{obs}(s), \delta s \rangle = \langle (\partial_c m)^* \nabla_m j_{obs}, w \rangle + \langle \partial_s \mathcal{L}(s, c(s)) \cdot \delta s, z \rangle + \langle \partial_c \mathcal{L}(s, c(s)) \cdot w, z \rangle$$

for any z

Taking Z solution of $\mathcal{L}(s, c(s))^* z + (\partial_c m)^* \nabla_m j_{obs} = 0$

$$\langle \nabla j_{obs}(s), \delta s \rangle = \langle \partial_s \mathcal{L}(s, c(s)) \cdot \delta s, Z \rangle$$

Sensors

Observations m^{obs}

Task 3.2.1: Optimization problem:

$$\begin{cases} \text{find } s_a(x, y, t) \text{ such that} \\ s_a(x, y, t) = \arg \min_{s(x, y, t)} (j_{obs}(s) + j_{reg}(s)) \end{cases}$$

with j_{reg} **a regularization term and**

$$j_{obs}(s) = \|m(c(s)) - m^{obs}\|_2^2.$$

Prediction of the observed variable $m(c(s))$.

Direct model

Sensors

Inverse problem
Control variable: pest density s .

Optimal source term s_a :

pest distribution that makes the **direct model** fits at best the observations m^{obs} .

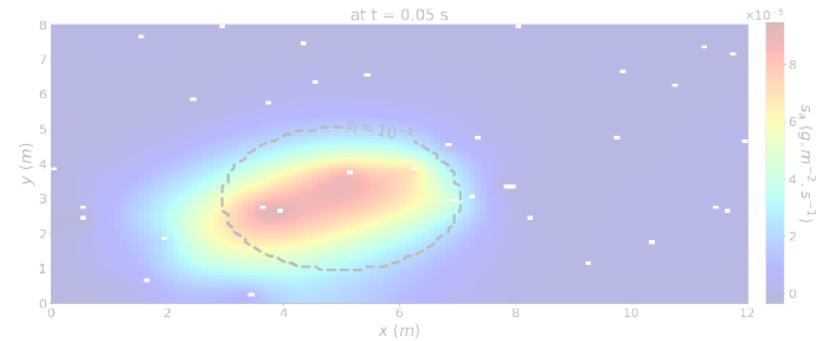


Figure: Optimal source term s_a and target s_t (dashed line). The white dots are the locations of the sensors.



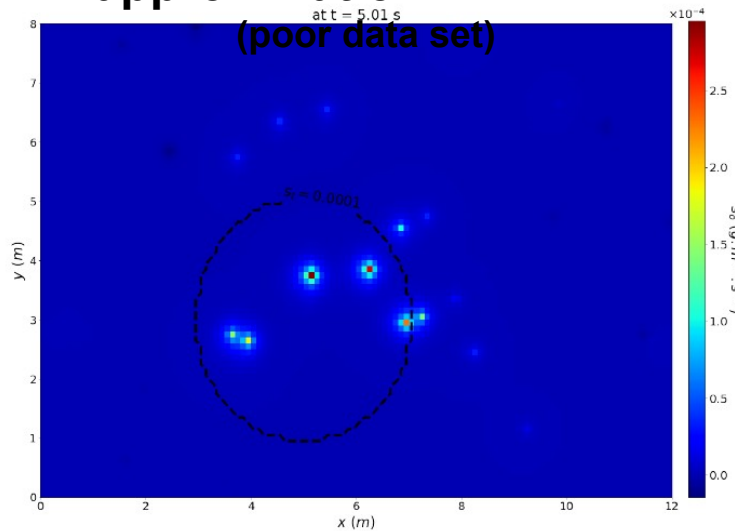
Regularization
term

Statistical characteristic: (sparsity, smoothness)

Biological characteristic :
- favorite pest habitat
(Thikonov)

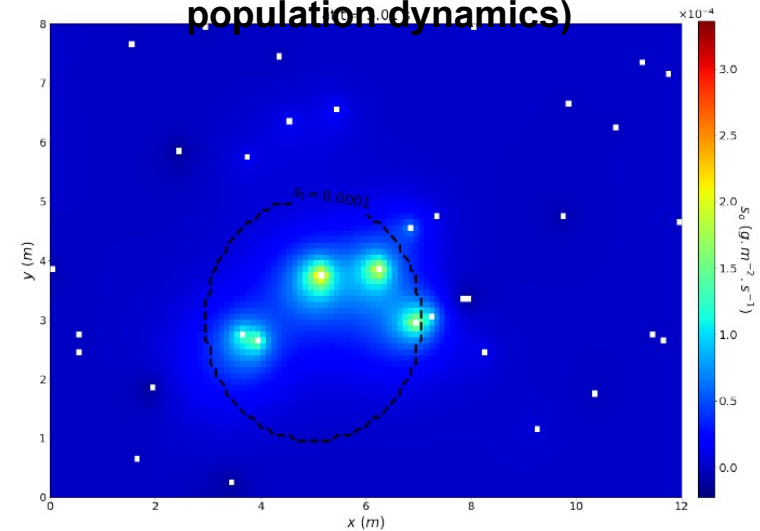
- population dynamic model

**Very bad
approximation**



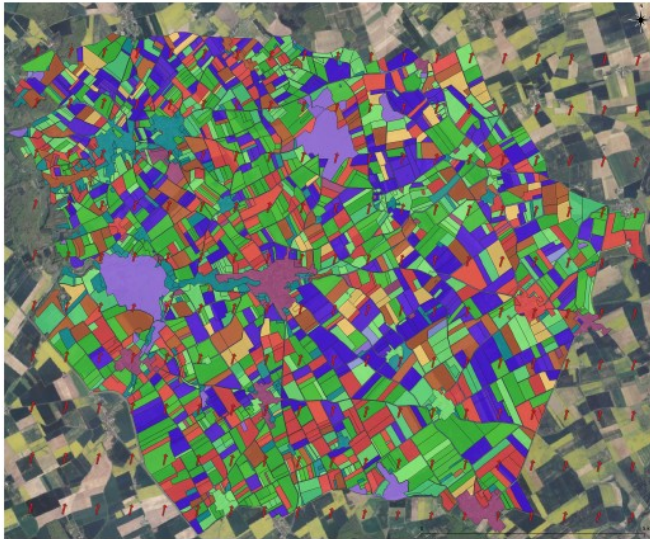
(a) no regularization

Enhanced approximation
(data supplemented by
population dynamics)

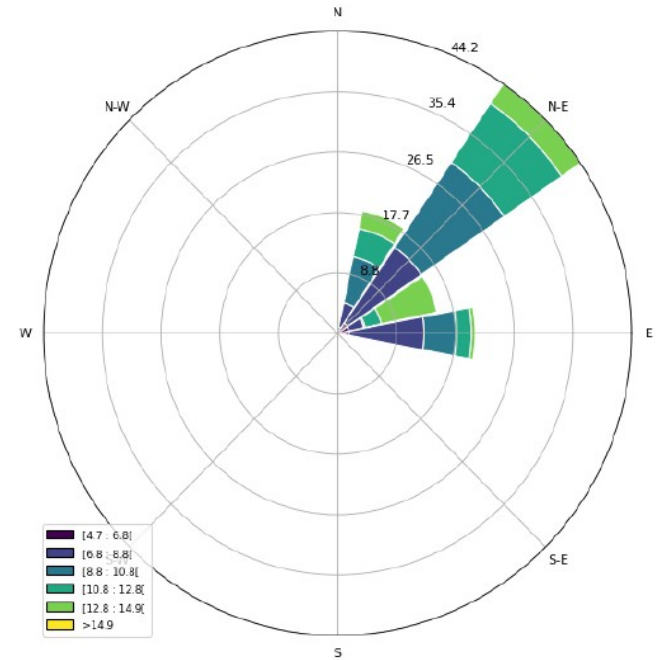


(b) stationnary population dynamique regularization

Application to a more realistic environmental conditions



(a) map of the test case with the land occupation and wind field at a given time

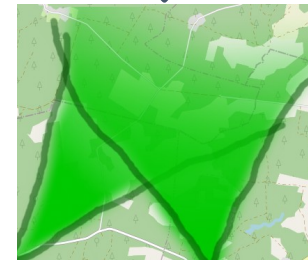
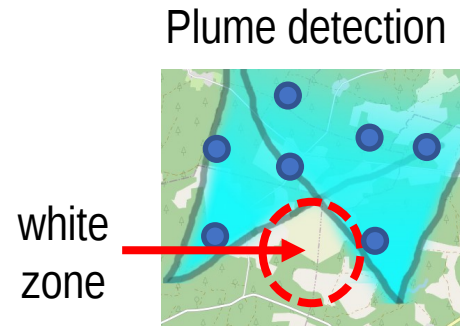
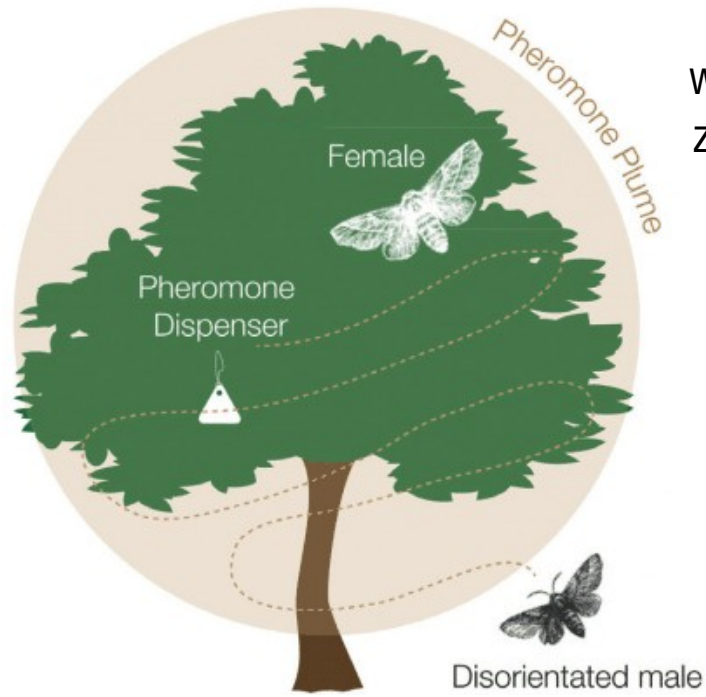


(b) wind rose over the test case and the whole time window (one day)

Figure: Test case around the village of Selommès between 16/11/2022 12:00 and 17/11/2022 12:00 (one day time window)

Related problem on “mating disruption optimization”

Detection of "white zones" during mating disruption



Optimal positioning of pheromone dispensers

Conclusion

- Inverse problem for pheromone emission source detection
- Variational inversion method
- Main work on regularization (biologically informed inverse problem)
- Practical issues (input data, sensor data, computing method)
- Additional maths : optimal placement, surrogate models, sensitivity analysis



Acknowledgements

- Philippe Lucas and collaborators
- Pherosensor WP3 colleagues
 - T.Malou
 - E.Vergu
 - B.Laroche
 - F.Deslandes
 - K.Adamczyk
 - N.Parisey

