Modeling the population dynamics of zoonoses, and the Allee effect

Cédric Chane Ki Chune
under the supervision of Guillaume Fournié,
Olivier Bonnefon, Lionel Roques

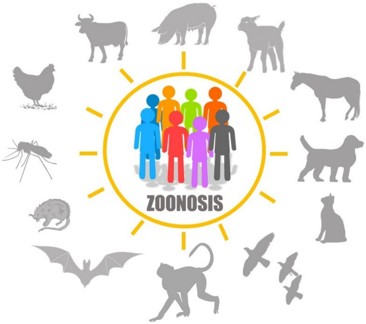
INRAE - Unité BioSP

May 16, 2024

Modeling zoonoses

Common characteristics:

- Spread between hosts (species, populations)
- Spread geographically,
- Mutate,
- ...



Two possible approaches: modeling the hosts vs. modeling the pathogen

Host-centered modeling

Example: compartmental models, like SIR

The base SIR model

$$\frac{dS}{dt} = -\beta IS \quad ; \quad \frac{dI}{dt} = \beta IS - \gamma I \quad ; \quad \frac{dR}{dt} = \gamma I$$

Host-centered modeling

Example: compartmental models, like SIR

The base SIR model

$$\frac{dS}{dt} = -\beta IS \quad ; \quad \frac{dI}{dt} = \beta IS - \gamma I \quad ; \quad \frac{dR}{dt} = \gamma I$$

Many extensions: V (vaccinated), D (deceased), C (carrier), E (exposed), ...

Host-centered modeling

Example: compartmental models, like SIR

The base SIR model

$$\frac{dS}{dt} = -\beta IS \quad ; \quad \frac{dI}{dt} = \beta IS - \gamma I \quad ; \quad \frac{dR}{dt} = \gamma I$$

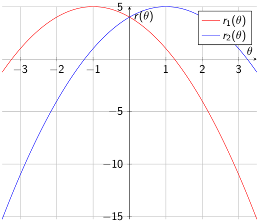
Many extensions: V (vaccinated), D (deceased), C (carrier), E (exposed), ...

- Directly models quantities relevant to public health
- Loses information about the pathogen (e.g. how it interacts with different hosts)

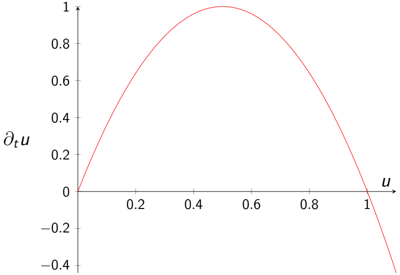
System of equations for a pathogen-centered model

$$\partial_t u_i = \underbrace{d\Delta_x u_i}_{\text{geographical diffusion}} + \underbrace{\mu\Delta_\theta u_i}_{\text{phenotypical diffusion}} + \underbrace{r_i(\theta)u_i}_{\text{fitness}} - \underbrace{\frac{u_i\rho_i}{K_i(x)}}_{\text{competition}} + \underbrace{\alpha K_i(x)\sum_{j\neq i} u_j}_{\text{contact with other hosts}}$$

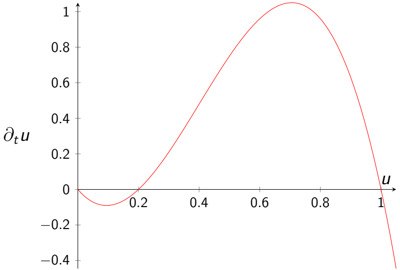
- $u_i(t,x,\theta)$: population density of the pathogen on host i,
- $\rho_i(t,x) = \int_\Theta u_i(t,x,\theta)d\theta$,
- $r_i(\theta)$: fitness of the pathogen on host i,
- $K_i(x)$: population density of host i.



Allee effect



(a) No Allee effect



(b) Strong Allee effect

It may arise due to random fluctuations, which affect the survival of a species more when there are fewer individuals.

System of equations with Allee effect

$$\partial_t u_i = d\Delta_x u_i + \mu\Delta_\theta u_i + r_i(\theta)u_i - \frac{u_i\rho_i}{K_i(x)} + \alpha K_i(x)\sum_{j\neq i} u_j + f(u_i) \quad (1)$$

$f(u_i)$ is such that u_i decreases when it is below a threshold ε.
When u_i is large, $f(u_i)$ is negligible.

System of equations with Allee effect

$$\partial_t u_i = d\Delta_x u_i + \mu\Delta_\theta u_i + r_i(\theta)u_i - \frac{u_i\rho_i}{K_i(x)} + \alpha K_i(x)\sum_{j\neq i} u_j + f(u_i) \quad (1)$$

$f(u_i)$ is such that u_i decreases when it is below a threshold ε.
When u_i is large, $f(u_i)$ is negligible.

A reassuring result

There exists a unique solution to the system (1).

System of equations with Allee effect

$$\partial_t u_i = d\Delta_x u_i + \mu\Delta_\theta u_i + r_i(\theta)u_i - \frac{u_i\rho_i}{K_i(x)} + \alpha K_i(x)\sum_{j\neq i} u_j + f(u_i) \quad (1)$$

$f(u_i)$ is such that u_i decreases when it is below a threshold ε.
When u_i is large, $f(u_i)$ is negligible.

A reassuring result

There exists a unique solution to the system (1).

Does adding the Allee effect make the model more useful?

An argument why:

Will a given pathogen persist or go extinct?

Without the Allee effect: the answer depends only on the parameters and not on the initial sample i.e. if a pathogen can persist, it will always do so, not matter how small the starting population is.

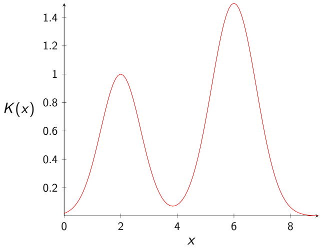


Figure: A host population concentrated in two locations. In the model without the Allee effect, if the pathogen spreads in one location, it will also spread in the other.

The model with the Allee effect fixes those problems: small starting populations go extinct and spread can be contained.

Useful for accurately testing the efficacy of public health measures.

The model with the Allee effect fixes those problems: small starting populations go extinct and spread can be contained.

Useful for accurately testing the efficacy of public health measures.

What's next?

- Do stochastic models recover the same consequences?
- Combine the host-centered and pathogen-centered models,
- Develop a framework for simulation and parameter estimation,
- Test the models against real datasets.

The model with the Allee effect fixes those problems: small starting populations go extinct and spread can be contained.

Useful for accurately testing the efficacy of public health measures.

What's next?

- Do stochastic models recover the same consequences?
- Combine the host-centered and pathogen-centered models,
- Develop a framework for simulation and parameter estimation,
- Test the models against real datasets.

Thank you for your attention

(contact: *cedric.chane-ki-chune@inrae.fr*)