Mathematical modelling for sustainable crop protection

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Crop protection

SUSTAINABLE DEVELOPMENT GOALS

- Population & food demand are increasing
  "By 2050, global agricultural production must increase by 70% [...] to meet the demand from a population of 9 billion" [FAO]
- Crop pests, diseases and weeds threaten food security
  20–40% of crop yields destroyed every year
- Agriculture is a major sector for employment and revenues in many (developing) countries
  nearly 80% of working poor live in rural areas [FAO]

➥ Controlling crop pests is a major issue

2 ZERO HUNGER

1 NO POVERTY

3 GOOD HEALTH AND WELL-BEING

- Chemical pesticides:
  - negative impact on human health & the environment
  - variable effectiveness, induce pest resistance
  - high financial and labour costs

➥ Need for sustainable control methods

15 LIFE ON LAND

12 RESPONSIBLE CONSUMPTION AND PRODUCTION

Alternatives to chemical pesticides

- Physical methods: traps, soil solarisation...
- Cultural practices: rotation, strip-cropping, destruction of residues, fallow, stump pruning...
- Biocontrol agent releases:
  - biopesticides: micro-organisms (bacteria, fungi, viruses), bio-derived chemicals (pheromones...)
  - macro-organisms: predators, parasites
  - sterile insect technique
- Plant resistance deployment:
  - qualitative (gene-for-gene / complete)
  - quantitative (polygenic / partial) resistance
  - plant tolerance

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Castillo & Infante

Epidemiological modelling

Compartmental models to represent the dynamics (progression over time) of an infectious disease in a population, where individuals are:

Susceptible = healthy, naive
Exposed = infected, latent, non infectious
Infected = infectious
Recovered = immune, resistant / removed

E.g. SEIRS model

$$\begin{cases} \dot{S} = \\ \dot{E} = \\ \dot{I} = \\ \dot{R} = \end{cases}$$

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force of infection βI

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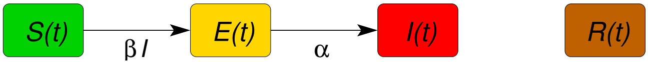

$$\begin{cases} \dot{S} = -\beta I S \\ \dot{E} = \beta I S - \alpha E \\ \dot{I} = \alpha E \\ \dot{R} = \end{cases}$$

latency period $1/\alpha$

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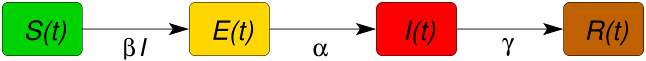

$$\begin{cases} \dot{S} = \; -\beta I\, S \\ \dot{E} = \beta I\, S - \alpha E \\ \dot{I} = \alpha E - \gamma I \\ \dot{R} = \gamma I \end{cases}$$

duration of infectiousness $1/\gamma$

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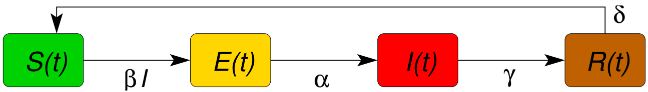

$$\begin{cases} \dot{S} = -\beta I\, S + \delta R \\ \dot{E} = \beta I\, S - \alpha E \\ \dot{I} = \alpha E - \gamma I \\ \dot{R} = \gamma I - \delta R \end{cases}$$

duration of immunity $1/\delta$

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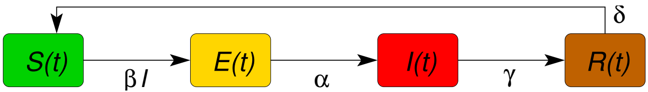

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Constant population: $P = S(t) + E(t) + I(t) + R(t)$

Equilibria ($\dot{S} = \dot{E} = \dot{I} = \dot{R} = 0$):

- disease-free (DFE): $S^* = P$, $E^* = I^* = R^* = 0$
- endemic (with disease) if $\gamma < \beta P$

Basic reproduction number $\mathcal{R}_0$

Number of secondary cases generated by an average index case during its entire infectious period, when introduced in a fully susceptible population

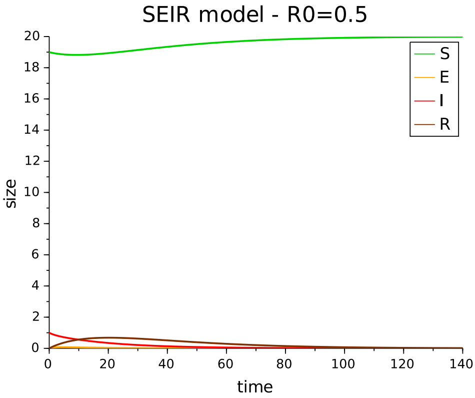
$\mathcal{R}_0$ is a threshold (DFE local stability)

- $\mathcal{R}_0 < 1 \rightarrow$ no epidemic, infection cannot settle in
- $\mathcal{R}_0 > 1 \rightarrow$ epidemic

E.g. SEIRS model

$$\mathcal{R}_0 = \frac{\beta P}{\gamma}$$

If $\mathcal{R}_0 < 1$: stable DFE



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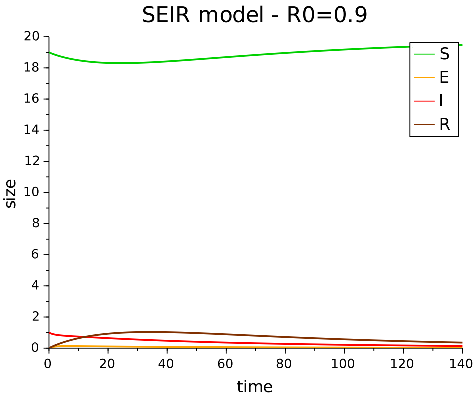

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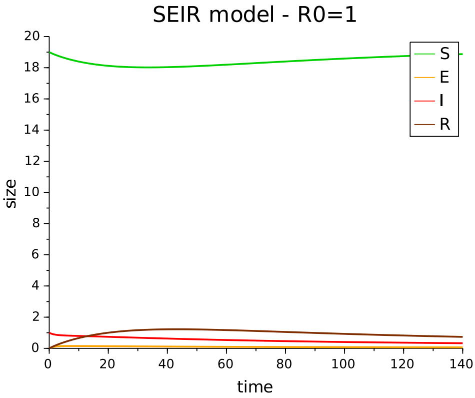

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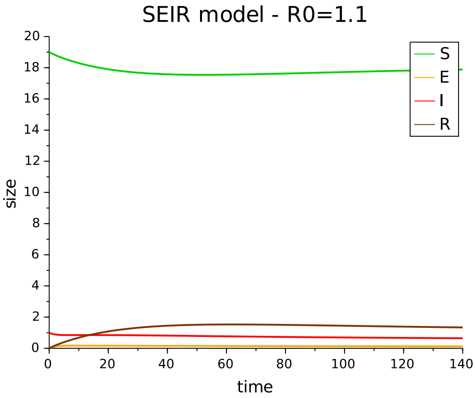
If $\mathcal{R}_0 > 1$:

- unstable DFE
- endemic equilibrium



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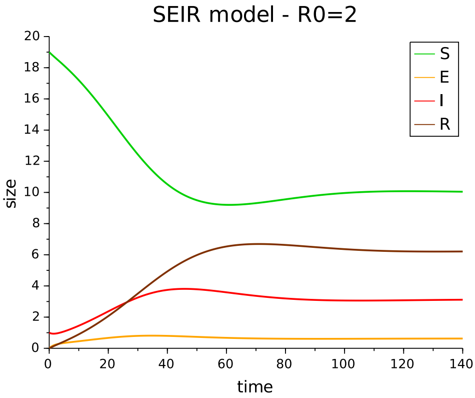

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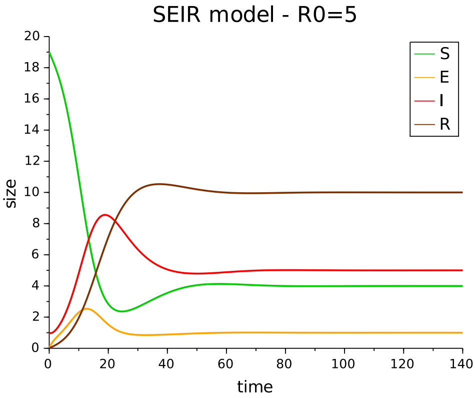

Plant epidemiology

Epidemiological models for human populations, but also animal and **plant** populations

Plant & crop specificities

- Definition of a (healthy/infected) individual: plant/tree, (part of) leaf, root, fruit...?

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- Plants affected by diseases and pests: grazers, phytophagous insects...
- Plants don't move: "contacts" via vectors, wind, water, free-living pathogen stages...
- Plants usually don't recover, but variable susceptibility
- Crops managed by humans: planting, harvest, partial environmental control...
- Seasonality plays an important role in annual & perennial crops

Approach

Design and analyse **epidemiological models** to:

- better understand plant–parasite interactions
- predict the evolution of damages
- provide efficient and sustainable control strategies to limit damages and crop losses

Tools: **optimisation** and **control** theory

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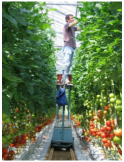
Different pathosystems

- single or multiple cropping seasons
- spatial scale:

A. Canaud

plant



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greenhouse

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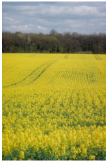
under cover



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field

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landscape

1. Optimising cultural practices – Banana burrowing nematodes

PhD (2021): **Israël TANKAM CHEDJOU**

Frédéric GROGNARD, Jean Jules TEWA + Ludovic MAILLERET

2. Optimal biopesticide-based control – Coffee berry borer
3. Self-financing model for cabbage crops with pest management

Banana burrowing nematodes (*Radopholus similis*)


Rosendahl

A: [Jesus, Agron Sustain Dev 2014]; B: M. MacClure, Univ. Arizona; C: [Zhang, EJPP 2012]

- Banana, including plantain: major staple food – *Cameroon: 2% GDP*
- Burrowing nematodes develop, feed and reproduce in roots
- Severe crop losses (up to plant toppling)

- Control
  - chemical nematicides: harmful to environment and human health
  - cropping practices (soil sanitation)
  - biological control: limited options
  - tolerant or resistant banana cultivars

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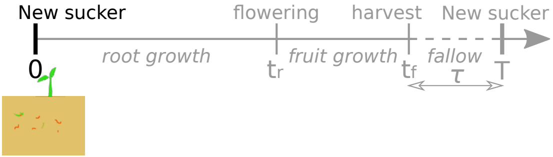
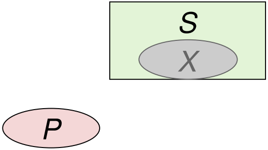
- Control
  - chemical nematicides: harmful to environment and human health
  - cropping practices (soil sanitation): fallow
  - biological control: limited options
  - tolerant or resistant banana cultivars

How best to implement fallows to limit pest damages and preserve yield?

Model: initialisation

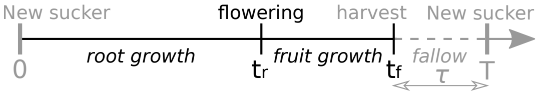


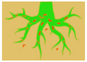


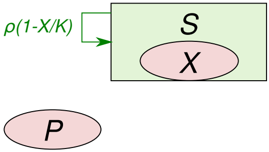
$$\begin{cases} S(0) = S_0 & \text{plant root} \\ X(0) = 0 & \text{nematodes in root} \\ P(0) = P_0 & \text{nematodes in soil} \end{cases}$$

Hypotheses:

- nursery-grown pest-free sucker (no asexual reproduction by offshoots)
- no male nematodes (not infective & not necessary for reproduction)



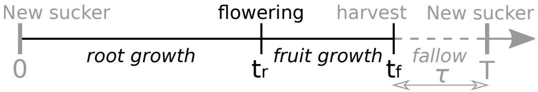


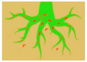


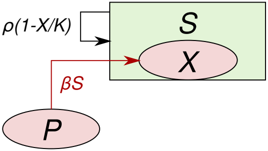
root growth

$$\begin{cases} \dot{S} = \rho(t)\, S(1 - \frac{S}{K}) \\ \dot{X} = \\ \dot{P} = \end{cases}$$

Model: cropping season



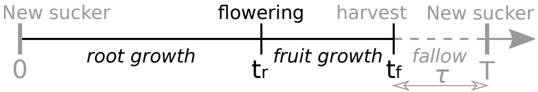


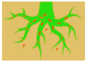


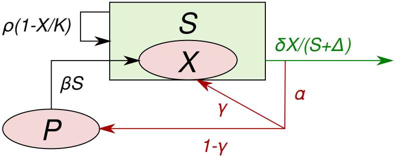
root growth

$$\begin{cases} \dot{S} = \rho(t)\, S\left(1 - \dfrac{S}{K}\right) \\ \dot{X} = \qquad + \beta\, P\, S \\ \dot{P} = \qquad - \beta\, P\, S \end{cases}$$

root entering



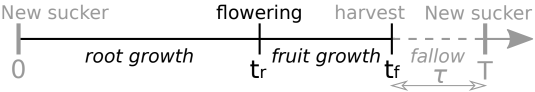


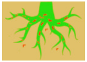


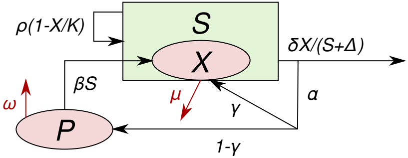
root growth — root consumption

$$\begin{cases} \dot{S} = \rho(t)\, S\left(1 - \dfrac{S}{K}\right) & - \delta \dfrac{S\,X}{S+\Delta} \\ \dot{X} = & + \beta\, P\, S \;+\; \delta \dfrac{S\,X}{S+\Delta}\, \alpha\, \gamma \\ \dot{P} = & - \beta\, P\, S \;+\; \delta \dfrac{S\,X}{S+\Delta}\, \alpha\, (1-\gamma) \end{cases}$$

root entering — feeding & reproduction



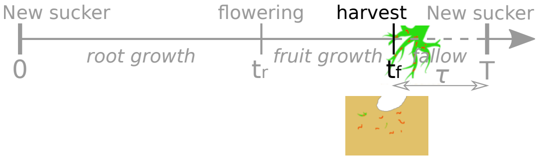


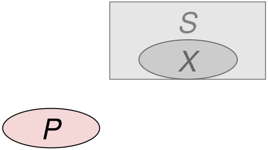


root growth — root consumption

$$\begin{cases} \dot{S} = \rho(t)\, S(1 - \frac{S}{K}) & - \delta \frac{S\,X}{S+\Delta} \\ \dot{X} = & + \beta P S + \delta \frac{S\,X}{S+\Delta} \alpha \gamma & - \mu X \\ \dot{P} = & - \beta P S + \delta \frac{S\,X}{S+\Delta} \alpha (1-\gamma) & - \omega P \end{cases}$$

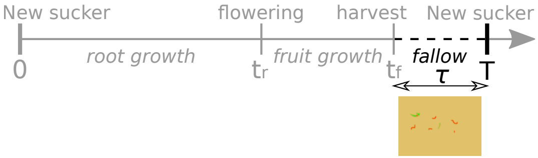
root entering — feeding & reproduction — mortality

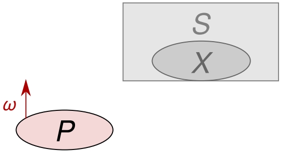




$$\begin{cases} S(t_f^+) = 0 \\ X(t_f^+) = 0 \\ P(t_f^+) = P(t_f) + q\,X(t_f) \end{cases}$$

Hypothesis: some infested roots remain in soil at uprooting

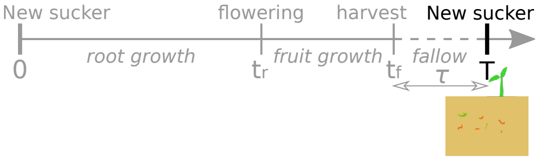


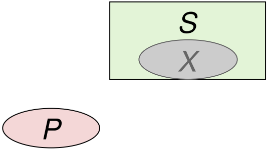


$$\begin{cases} \dot{S} = 0 \\ \dot{X} = 0 \\ \dot{P} = -\omega\, P \end{cases}$$

Hypothesis: no alternative hosts for nematodes during fallow

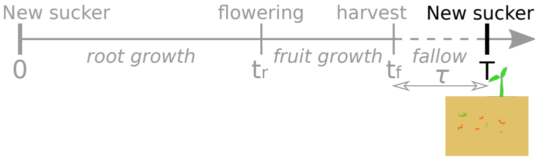
Model: new sucker

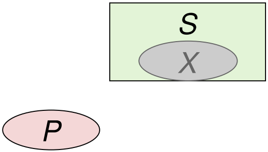




$$\begin{cases} S(T^+) = S_0 \\ X(T^+) = 0 \\ P(T^+) = P(T) = \big(P(t_f) + q\, X(t_f)\big)\, e^{-\omega\, \tau} \end{cases}$$

Hypothesis: new nursery-grown pest-free sucker





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$$

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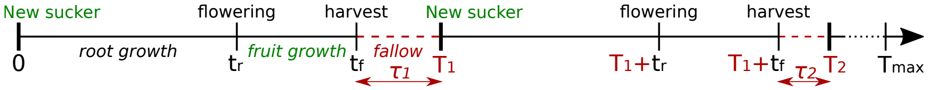
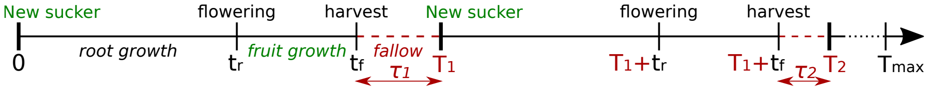
Etc. for the next seasons

Optimal fallow deployment



- Seasonal yield proxy: $Y_1 = \int_{t_r}^{t_f} w\, S(t) dt$
- Cost of a pest-free sucker: c
- Seasonal profit: $R_1 = Y_1 - c$

Optimal fallow deployment



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- Cost of a pest-free sucker: c
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Optimisation problem

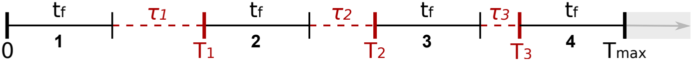
Determine the number and duration of fallow periods (τ_i) which
maximise the cumulated profit on a fixed multi-seasonal time horizon ($T_{\max}$):

$$\max_{N,\tau_i} \sum_{i=1}^{N} R_i(\tau_{j,\, j<i})$$

Numerical method: Adaptive Random Search algorithm

Optimal fallow deployment

Admissible fallows (τ_i) such that last cropping season ends at T_{max}, e.g.:



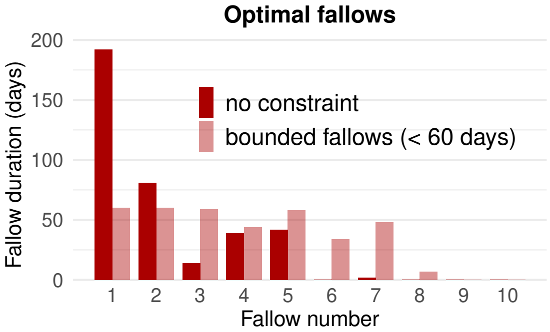
Maximum number of fallows: $n_{\max} = \left\lfloor \frac{T_{\max}}{t_f} \right\rfloor - 1$

1. For $n = 1, \ldots, n_{\max}$

   optimisation over n-simplex: $\sum_{i=1}^{n} \tau_i = T_{\max} - (n+1)\, t_f$

   $\rightarrow$ optimal profit $R^{n\star}$

2. Select highest $R^{n\star}$



Example for $T_{max} = 4000$ days ($n_{\max} = 11$)

In a nutshell

How best to implement fallows to limit pest damages and preserve yield?

Fallows can limit nematode infestation and maintain profit

– especially with long fallows early on to sanitise the soil
– but expensive pest-free suckers → follow-up with fallows and natural reproduction

I. Tankam Chedjou et al., 2021. *Applied Mathematics and Computation* 397:125883.
doi: 10.1016/j.amc.2020.125883

I. Tankam Chedjou et al., 2021. *Journal of Interdisciplinary Methodologies and Issues in Science* 8 – Digital Agriculture in Africa. doi: 10.18713/JIMIS-120221-8-4

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In terms of behavioural epidemiology?

- basic economic criterion
- long term optimisation of cultural practices, but open-loop

One step further: infestation feedback to represent the grower's decision (fallow or not fallow, etc.) each season?

1. Optimising cultural practices – Banana burrowing nematodes

2. Optimal biopesticide-based control – Coffee berry borer

   PhD (2022): **Yves FOTSO FOTSO**

   

   Samuel BOWONG, Frédéric GROGNARD, Berge TSANOU

3. Self-financing model for cabbage crops with pest management

Coffee berry borers (*Hypothenemus hampei*)

Uccao Cameroun

[Burbano, JIS 2011]

- Coffee: cash crop for tropical developing countries – *25 million households [FAO]*
- CBB: mostly develop and feed in coffee berries
- In all production countries, economic losses $>$ 500 million \$/year

- Control
  - chemical insecticides: poorly efficient (cryptic pest)
  - trapping
  - cropping practices: strip-picking, stump pruning
  - biological control: parasitoid or predator insects, entomopathogenic fungi

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  - cropping practices: strip-picking, stump pruning
  - biological control: parasitoid or predator insects, entomopathogenic fungi (*Beauveria bassiana*)

A. Ramírez

How best to apply a biopesticide to control CBB during a growing season?

$\dot{s} =$ healthy berries

$\dot{i} =$ infested berries

$\dot{y} =$ colonising ♀ (♂ not limiting)

$\dot{z} =$ infesting ♀

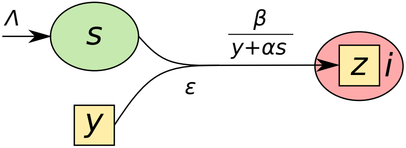
new berries

$\dot{s} = \quad \Lambda$ healthy berries

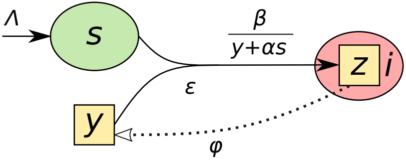
$\dot{i} =$ infested berries

$\dot{y} =$ colonising ♀ (♂ not limiting)

$\dot{z} =$ infesting ♀

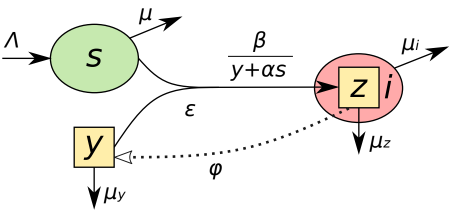


$$\begin{aligned}
&\ \text{new berries} \qquad \text{infestation} \\
\dot{s} &= \Lambda - \beta\frac{sy}{y+\alpha s} && \text{healthy berries} \\
\dot{i} &= + \beta\frac{sy}{y+\alpha s} && \text{infested berries} \\
\dot{y} &= - \varepsilon\beta\frac{sy}{y+\alpha s} && \text{colonising ♀ (♂ not limiting)} \\
\dot{z} &= + \varepsilon\beta\frac{sy}{y+\alpha s} && \text{infesting ♀}
\end{aligned}$$

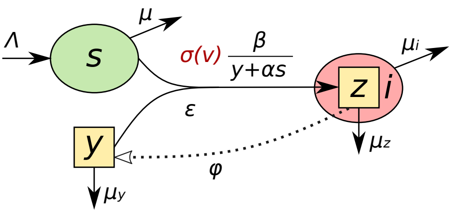


	new berries		infestation	
$\dot{s} =$	Λ	$-$	$\beta \dfrac{sy}{y + \alpha s}$	healthy berries
$\dot{i} =$		$+$	$\beta \dfrac{sy}{y + \alpha s}$	infested berries
	emergence			
$\dot{y} =$	φz	$-$	$\varepsilon\beta \dfrac{sy}{y + \alpha s}$	colonising ♀ (♂ not limiting)
$\dot{z} =$		$+$	$\varepsilon\beta \dfrac{sy}{y + \alpha s}$	infesting ♀

Single season model



	new berries / emergence		infestation	mortality	
$\dot{s} =$	Λ	$-$	$\beta \dfrac{sy}{y+\alpha s}$	$-\mu s$	healthy berries
$\dot{i} =$		$+$	$\beta \dfrac{sy}{y+\alpha s}$	$-\mu_i i$	infested berries
$\dot{y} =$	φz	$-$	$\varepsilon\beta \dfrac{sy}{y+\alpha s}$	$-\mu_y y$	colonising ♀ (♂ not limiting)
$\dot{z} =$		$+$	$\varepsilon\beta \dfrac{sy}{y+\alpha s}$	$-\mu_z z$	infesting ♀



new berries — infestation — mortality

$$\dot{s} = \Lambda \; - \sigma(v)\; \beta \frac{sy}{y + \alpha s} \; -\mu s$$ healthy berries

$$\dot{i} = \; + \sigma(v)\; \beta \frac{sy}{y + \alpha s} \; - \mu_i i$$ infested berries

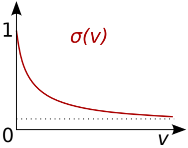
emergence

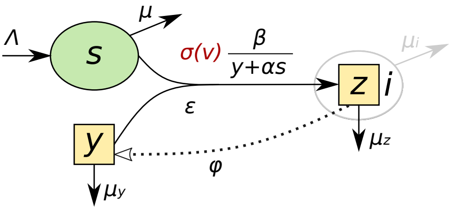
$$\dot{y} = \varphi z \; - \sigma(v) \varepsilon \beta \frac{sy}{y + \alpha s} \; - \mu_y y$$ colonising ♀ (♂ not limiting)

$$\dot{z} = \; + \sigma(v) \varepsilon \beta \frac{sy}{y + \alpha s} \; - \mu_z z$$ infesting ♀

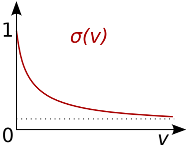
$$\dot{v} = h(t) \; -\gamma v$$ fungus load

control — decay





	new berries		infestation	mortality	
$\dot{s} =$	Λ	$-\,\sigma(v)$	$\beta \dfrac{sy}{y+\alpha s}$	$-\mu s$	healthy berries
$\dot{i} =$		$+\,\sigma(v)$	$\beta \dfrac{sy}{y+\alpha s}$	$-\,\mu_i i$	infested berries $\simeq$ colonising ♀
	emergence				
$\dot{y} =$	φz	$-\,\sigma(v)\varepsilon\beta$	$\dfrac{sy}{y+\alpha s}$	$-\,\mu_y y$	colonising ♀ (♂ not limiting)
$\dot{z} =$		$+\,\sigma(v)\varepsilon\beta$	$\dfrac{sy}{y+\alpha s}$	$-\,\mu_z z$	infesting ♀
$\dot{v} =$	$h(t)$	$-\gamma v$			fungus load
	control	decay			



Optimal control

Optimal control problem

Determine the entomopathogenic fungus application $h(t)$

maximising the yield at the end of the cropping season $s(t_f)$,

:

$$\mathcal{J}(h) = \underbrace{\zeta_s\, s(t_f)}_{\text{yield}}$$

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Determine the entomopathogenic fungus application $h(t)$

maximising the yield at the end of the cropping season $s(t_f)$,

while **minimising the control cost** ($\leftrightarrow$ maximise profit)

:

$$\mathcal{J}(h) = \underbrace{\zeta_s\, s(t_f)}_{\text{yield}} - \underbrace{\int_0^{t_f} C\, h(t)\, dt}_{\text{cost}}$$

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& **the CBB population** for the next growing season $y(t_f)$:

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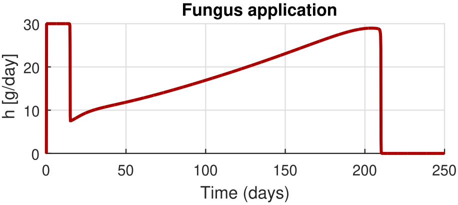
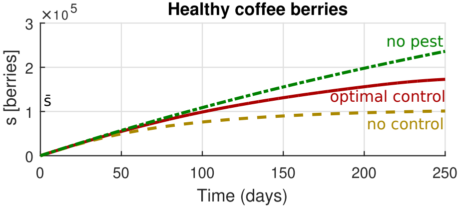
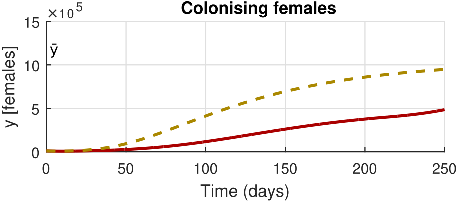
Pontryagin's Maximum Principle: **bang-singular-bang** solution

Numerical method: **BOCOP**







➥ Efficient biopesticide control:
- CBB population halved
- penalised profit $\mathcal{J}$ doubled

How to best apply a biopesticide to control CBB during a growing season?

– Optimal control gives a rough idea of how to apply pest control: start high
– Extension with 2 controls: biopesticide + traps

Y. Fotso Fotso et al., 2021. *Mathematical Methods in the Applied Sciences* 44(18):14569–14592. doi: 10.1002/mma.7726

Y. Fotso Fotso et al., 2023. *Journal of Optimization Theory and Applications* 196(3):882–899. doi: 10.1007/s10957-022-02151-7

A. Ramírez

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A. Ramírez

In terms of behavioural epidemiology?

- basic economic criterion
- optimal control(s), but open-loop and short term

Some steps further:
– *feedback for grower's decision*
– *information on local/regional prevalence*
– *risk perception to determine between-season controls (strip-picking, etc.)*

1. Optimising cultural practices – Banana burrowing nematodes

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3. Self-financing model for cabbage crops with pest management

Ongoing PhD: **Aurelien KAMBEU YOUMBI**

Frédéric GROGNARD, Berge TSANOU

Diamondback moth (*Plutella xylostella*)

Andrew Weeks

Alton N. Sparks, Jr

- Cabbages (*Brassica oleracea*): important staple food and source of income for smallholder farmers
- DBM: cosmopolitan insect, whose larvae graze mostly on cabbage plants
- Major pest, especially in regions with mild winters

- Control
  - chemical pesticides ⇒ moth resistance – botanical pesticides
  - cultural practices: inter-cropping, rotation...
  - biological control: parasitoid wasps

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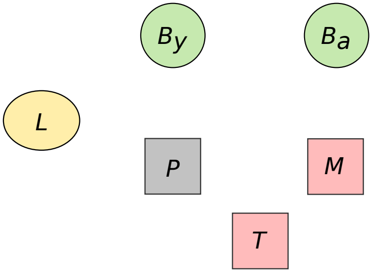
How should a smallholder farmer best use the cabbage crop revenues?

Self-financing model

Crop: B_y young biomass (susceptible) & B_a adult biomass (resistant)
Pest: L larva population (fast life cycle)
Pest control: P botanical pesticide (with antifeedant effect)
Money: M plantation current account & T grower's cumulated earnings



$$\dot{B}_y =$$

$$\dot{B}_a =$$

$$\dot{L} =$$

$$\dot{P} =$$

$$\dot{M} =$$

$$\dot{T} =$$

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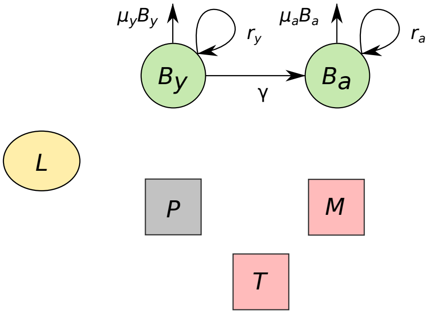

$$\dot{B}_y = r_y B_y - \mu_y B_y^2 - \gamma B_y$$

cabbage growth & ageing

$$\dot{B}_a = r_a B_a - \mu_a B_a^2 + \gamma B_y$$

$$\dot{L} =$$

$$\dot{P} =$$

$$\dot{M} =$$

$$\dot{T} =$$

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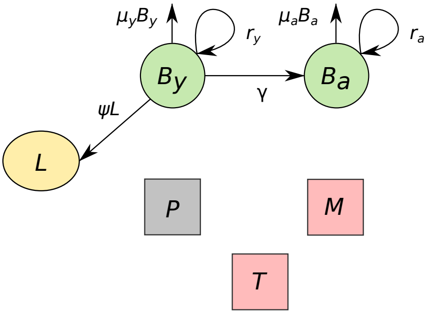

grazing

$$\dot{B}_y = r_y B_y - \mu_y B_y^2 - \gamma B_y - \frac{\psi L B_y}{1}$$

cabbage growth & ageing

$$\dot{B}_a = r_a B_a - \mu_a B_a^2 + \gamma B_y$$

recruitment

$$\dot{L} = \qquad + c'_B \frac{\psi L B_y}{1}$$

$$\dot{P} =$$

$$\dot{M} =$$

$$\dot{T} =$$

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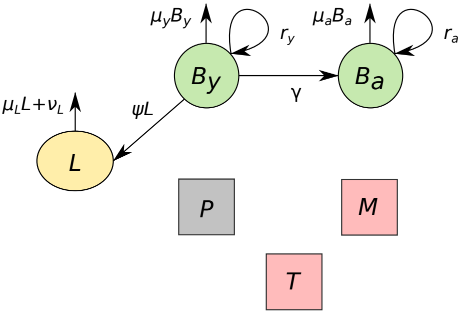

$$\dot{B}_y = \underbrace{r_y B_y - \mu_y B_y^2 - \gamma B_y}_{\text{cabbage growth \& ageing}} - \overbrace{\frac{\psi L B_y}{1}}^{\text{grazing}}$$

$$\dot{B}_a = r_a B_a - \mu_a B_a^2 + \gamma B_y$$

$$\dot{L} = \overbrace{-\nu_L L - \mu_L L^2}^{\text{larva mortality}} \qquad + \overbrace{c'_B \frac{\psi L B_y}{1}}^{\text{recruitment}}$$

$$\dot{P} =$$

$$\dot{M} =$$

$$\dot{T} =$$

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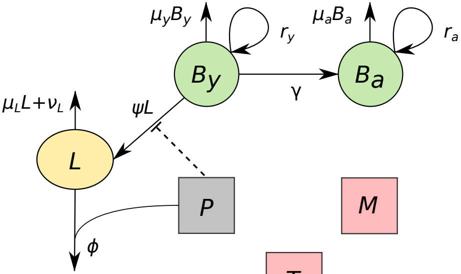

$$\dot{B}_y = \underbrace{r_y B_y - \mu_y B_y^2 - \gamma B_y}_{\text{cabbage growth \& ageing}} - \overbrace{\frac{\psi L B_y}{bP+1}}^{\text{grazing}}$$

$$\dot{B}_a = r_a B_a - \mu_a B_a^2 + \gamma B_y$$

$$\dot{L} = \overbrace{-\nu_L L - \mu_L L^2}^{\text{larva mortality}} - c_L \phi P L + \overbrace{c_B' \frac{\psi L B_y}{bP+1}}^{\text{recruitment}}$$

$$\dot{P} = \overbrace{-\phi P L}^{\text{larva uptake}}$$

$$\dot{M} =$$

$$\dot{T} =$$

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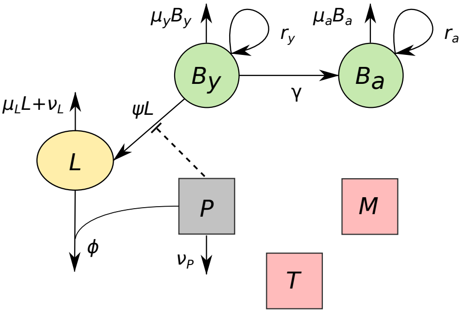

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$$\dot{P} = \overbrace{-\phi P L}^{\text{larva uptake}} \overbrace{- \nu_P P}^{\text{decay}}$$

$$\dot{M} =$$

$$\dot{T} =$$

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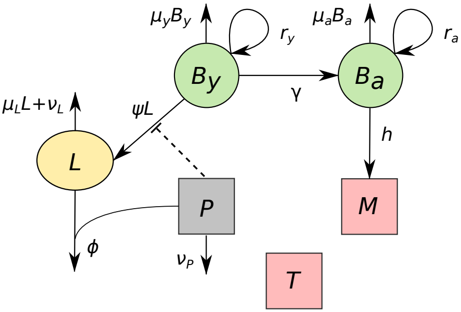

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$$\dot{B}_a = r_a B_a - \mu_a B_a^2 + \gamma B_y - \overbrace{h B_a}^{\text{harvest}}$$

$$\dot{L} = \overbrace{-\nu_L L - \mu_L L^2}^{\text{larva mortality}} - c_L \phi P L + \overbrace{c_B' \frac{\psi L B_y}{bP+1}}^{\text{recruitment}}$$

$$\dot{P} = \overbrace{-\phi P L}^{\text{larva uptake}} \overbrace{- \nu_P P}^{\text{decay}}$$

$$\dot{M} = \overbrace{c_B h B_a}^{\text{revenue}}$$

$$\dot{T} =$$

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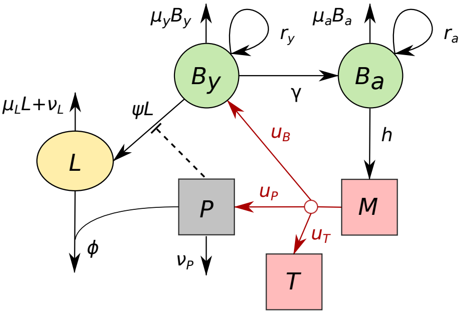

$$\dot{B}_y = r_y B_y - \mu_y B_y^2 - \gamma B_y - \overset{\text{grazing}}{\frac{\psi L B_y}{bP+1}} + c_M u_B$$

cabbage growth & ageing

$$\dot{B}_a = r_a B_a - \mu_a B_a^2 + \gamma B_y - \overset{\text{harvest}}{h B_a}$$

$$\dot{L} = \overset{\text{larva mortality}}{-\nu_L L - \mu_L L^2} - c_L \phi P L + \overset{\text{recruitment}}{c'_B \frac{\psi L B_y}{bP+1}}$$

$$\dot{P} = \overset{\text{larva uptake}}{-\phi P L} - \overset{\text{decay}}{\nu_P P} + c'_M u_P$$

$$\dot{M} = \overset{\text{revenue}}{c_B h B_a} - (u_B + u_P + u_T)$$

$$\dot{T} = u_T$$

Automatic controls: u_B new seedlings, u_P protection costs, and u_T net income

Self-financing model

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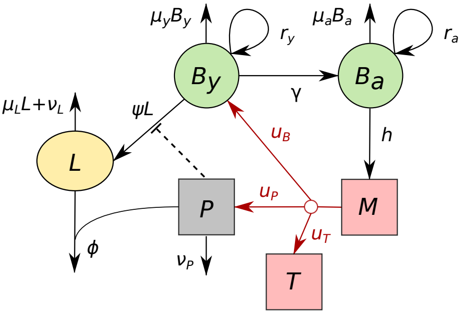

$$\dot{B}_y = r_y B_y - \mu_y B_y^2 - \gamma B_y - \underbrace{\frac{\psi L B_y}{bP+1}}_{\text{grazing}} + c_M u_B$$

cabbage growth & ageing

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$$\dot{M} = \underbrace{c_B h B_a}_{\text{revenue}} - (u_B + u_P + u_T)$$

$$\dot{T} = u_T$$

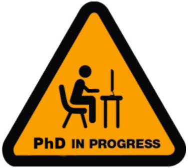
Automatic controls: u_B new seedlings, u_P protection costs, and u_T net income

A. Kambeu Youmbi et al., 2024. Preprint. HAL Id: hal-04589904

Optimisation

Optimisation problem

Determine controls u_B new seedlings, u_P protection costs, and u_T net income to **maximise the total earnings**, i.e. the final T.



- Static optimisation
- Open-loop or feedback controls
- Discrete controls

Conclusion

- **Modelling crop pests and diseases + ecofriendly control strategies**
  → insights in sustainable control deployment
- Some (necessary) simplifications: no abiotic factors, single pest, open-loop control...
- A few challenges (still):
  - "small data" → link with remote sensing?
  - growers' decisions → behavioural epidemiology?

Conclusion

- **Modelling crop pests and diseases + ecofriendly control strategies**
  → insights in sustainable control deployment
- Some (necessary) simplifications: no abiotic factors, single pest, open-loop control...
- A few challenges (still):
  - "small data" → link with remote sensing?
  - growers' decisions → behavioural epidemiology?

Work ∈ EPITAG = EPIdemiological modelling and control for Tropical AGriculture

French & Cameroonian researchers and students, with a background in applied mathematics, and an interest in crop diseases

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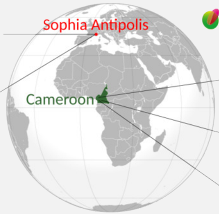

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More on EPITAG: https://team.inria.fr/epitag/