



# Evidence-based controls for epidemics using spatio-temporal stochastic models in a Bayesian framework

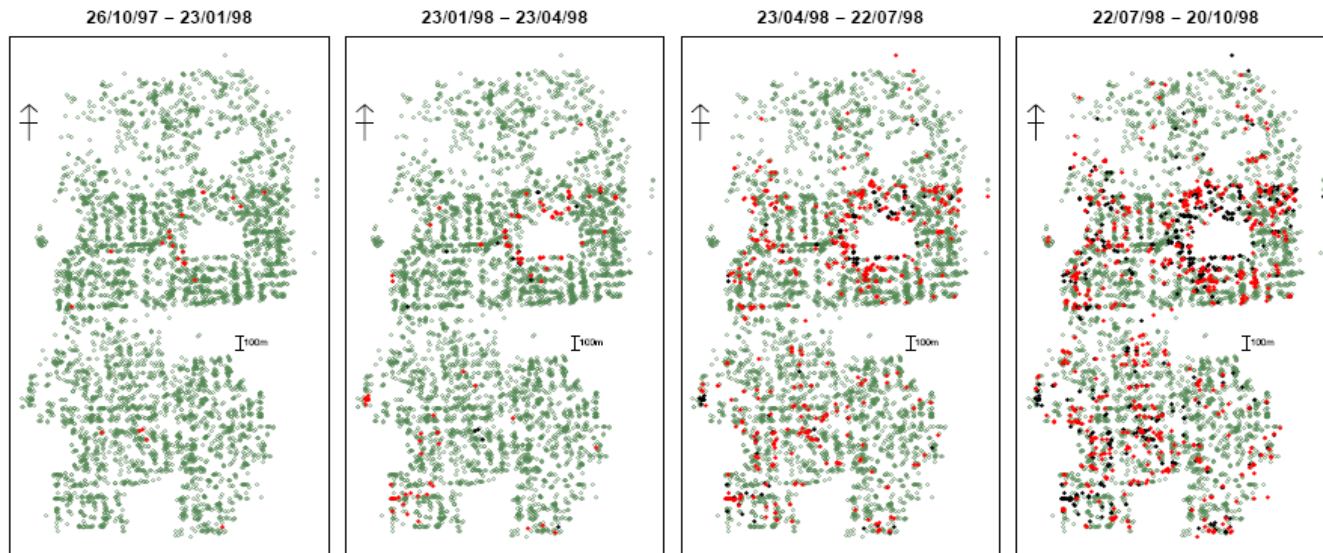
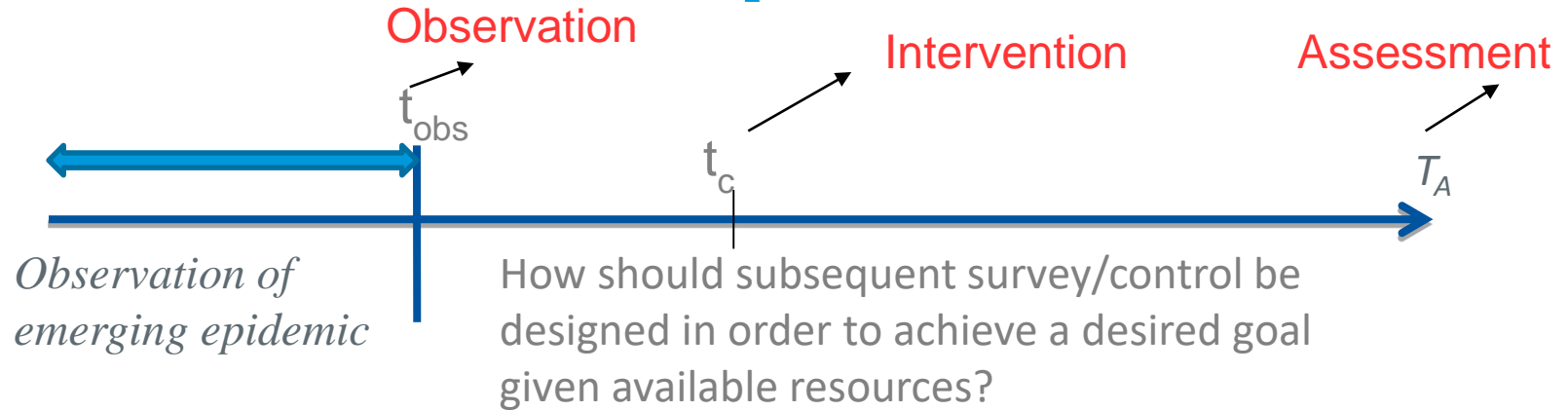
*Hola Adrakey*

JOINT WORK WITH: George Streftaris, Nik Cuniffe, Tim Gottwald, Chris Gilligan  
And Gavin Gibson

# Main messages

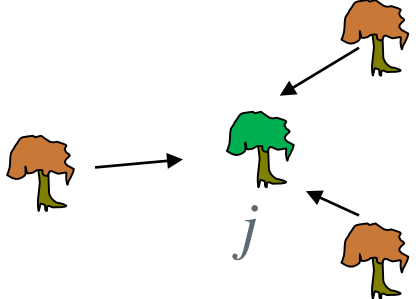
- Spatio-temporal stochastic models for informing control strategies
- Formulation of posterior measures for guiding control strategy
- Use of functional-model representations (non-centered parameterisations) for efficient comparison
- Conclusions – where to look?

# Generic problem



*Citrus canker epidemic: Dade County, Miami, Florida*

# SEIR spatio-temporal model

$S \rightarrow E$ : If  $j$  is in state  $S$  at time  $t$ , then 

$$\Pr(j \text{ exposed } (t, t+dt)) = (\varepsilon + \beta \sum_i K(d_{ij}, \alpha)) dt + o(dt)$$

$E \rightarrow I$ :  $T_E^j \sim \pi_{\theta_E}^E$  (random sojourn time in  $E$ )

$I \rightarrow R$ :  $T_I^j \sim \pi_{\theta_I}^I$  (random sojourn time in  $I$ )

**Parameters:**  $\theta = (\varepsilon, \beta, \alpha, \theta_I)$

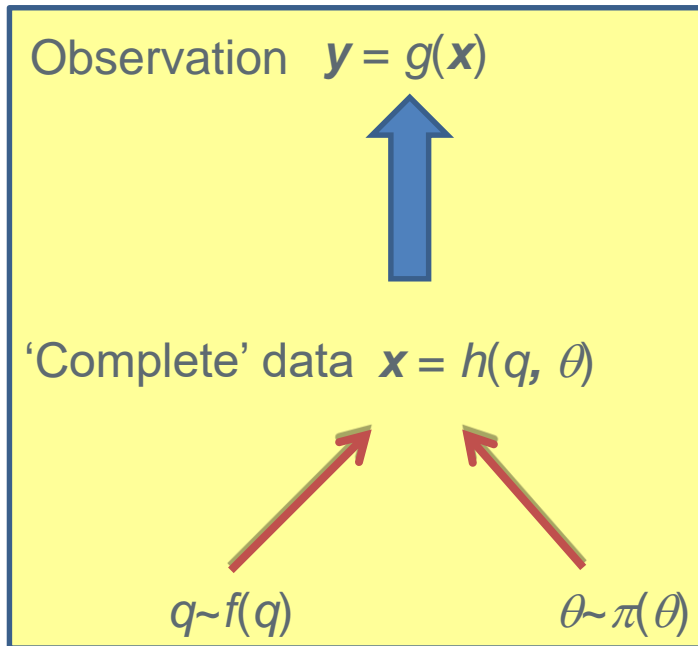
Here we focus on simpler SI model with cryptic infections – infections only become symptomatic after fixed (known) period  $\Delta$  (c.f. Neri et al (2014)).

# Model fitting in Bayesian framework

- For ‘complete’ data  $\mathbf{x}$  (e.g. times and nature of all transitions)  $\pi(\mathbf{x} | \theta)$  tractable
- Given censored/filtered/noisy data  $\mathbf{y}$ ,  $\pi(\mathbf{y} | \theta)$  typically intractable
- Use data augmentation and sample from 
$$\pi(\theta, \mathbf{x} | \mathbf{y}) \propto \pi(\theta) \pi(\mathbf{x}, \mathbf{y} | \theta)$$
 using e.g. MCMC
- Updating  $\mathbf{x}$  often requires reversible-jump techniques given variable dimension

(See e.g. GJG, 1997, O’Neill & Roberts, 1999, Streftaris & GJG, 2004, Forrester *et al.*, 2007, GJG *et al.*, 2006, Chis-Ster *et al.* 2008, Starr *et al.* 2009, Jewell & Roberts, 2007, Neri *et al.*, 2014, Lau *et al.*, 2015)

# Functional-model representations



Functional models (Dawid & Stone, 1983)

Consider outcome as deterministic function  $h(q, \theta)$  where  $q$  has known distribution independent of  $\theta$ .

In model choice  $q$  can be used as a latent residual process.

Investigating  $\pi(\theta, q | \mathbf{y})$  rather than  $\pi(\theta, \mathbf{x} | \mathbf{y})$  facilitates model assessment via latent classical tests.

Here we extend the idea to formulate models for epidemic dynamics in the presence of control  $\mathbf{d}$ , so that  $\mathbf{x} = h^*(q, \theta, \mathbf{d})$ .

# Sellke Construction (Sellke, 1983)

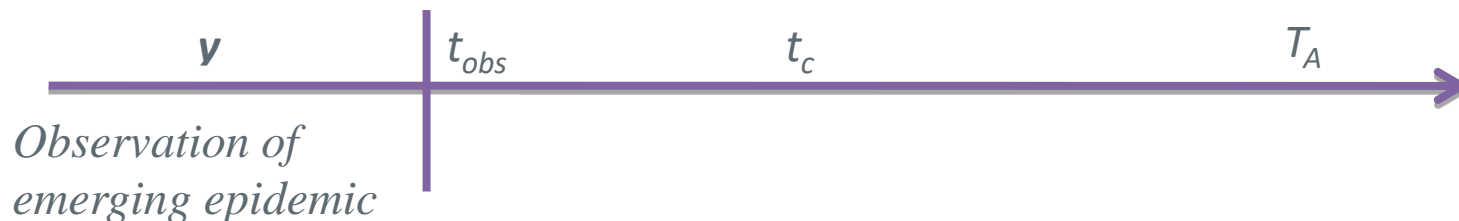
Assigns threshold  $q_i$  to each individual. If  $R_i(t)$  denotes infectious challenge to  $i$  at time  $t$ , infection time  $x_i$  occurs when integrated challenge reaches threshold

$$q_i = \int_0^{x_i} R_i(t) dt \sim \text{Exp}(1)$$

- Epidemic dynamics specified (for SI with cryptic) by  $q$  (vector of Sellke Thresholds) and  $\theta$ , i.e.  $(\mathbf{x} = h(q, \theta))$ .
- For controls  $\mathbf{d}$ , based on removal of infected individuals, it follows that  $\mathbf{x} = h^*(q, \theta, \mathbf{d})$ .
- Gives a means of coupling epidemic trajectories under different control strategies.

# Control strategies

- Based on removal of hosts found to be infected at control time  $t_c$ .
- $N'$  hosts can be targeted (resource constraint)
- Impact assessed at time  $t_A$  e.g. via number of infections occurring by  $t_A$ .





# Which hosts $j$ to target?

Based on  $E(G_M(\mathbf{x}(t), j) \mid \mathbf{y})$  at some time  $t > t_{obs}$

**Candidate measures** - ( $x_j$  denotes infection time of  $j$ )

□  $G_R(\mathbf{x}(t), j) = I_{\{x_j < t\}}$  - 'Risk'

□  $G_H(\mathbf{x}(t), j) = \sum_{x_i > t, i \neq j} \beta K(d_{ij}, \alpha)$  - 'Hazard'

□  $G_T(\mathbf{x}(t), j) = G_R(\mathbf{x}(t), j) \times G_H(\mathbf{x}(t), j)$  - 'Threat'

# Simulated epidemic 1

- Host population of size 1000 uniformly located over square region
- Simulate epidemic from SI model with:  
 $\alpha = 0.08, \beta = 7 \cdot 10^{-6}, \epsilon = 5 \cdot 10^{-5}$  and

$$K(d, \alpha) = \frac{1}{2\pi d\alpha} \exp(-d/\alpha)$$

- Observed data  $\mathbf{y}$ , snapshots of symptomatic sets at  $t=130, 160, 190, \dots, 460$
- Control applied at  $t_c=460, 470$
- Performance measure – reduction in posterior expectation of number of infections up to time  $t_A=500$  (relative to uncontrolled epidemic)

# Estimating expected reduction

- Use random sample from  $\pi(\theta, \mathbf{x}(t) \mid \mathbf{y})$  to generate sample of size  $m$  from  $\pi(\theta, Q \mid \mathbf{y})$ .
- Let  $u(x(T))$  denote the number of infections by time  $T$  for trajectory  $x(T)$ . Let  $d$  denote control strategy.

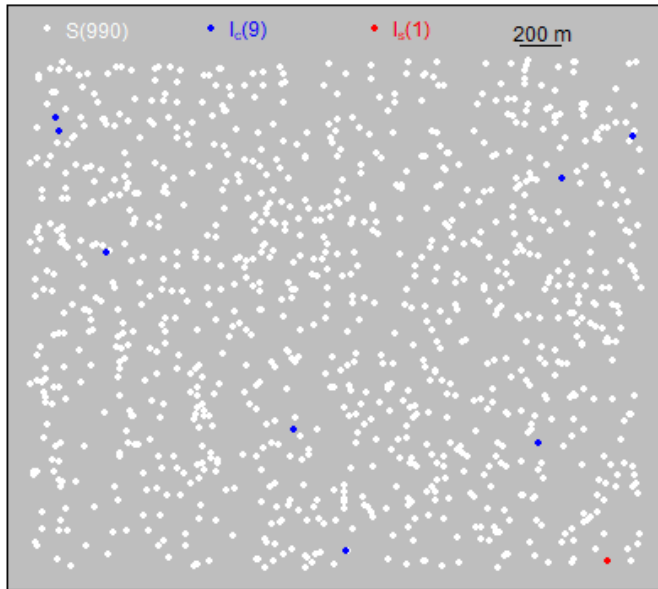
$$x(T) = h(\theta, Q), \quad x_d(T) = h^*(\theta, Q, d)$$

$$\text{EER}(d) = \frac{1}{n} \sum_i \{u(h^*(\theta_i, Q_i, d)) - u(h(\theta_i, Q_i))\}$$

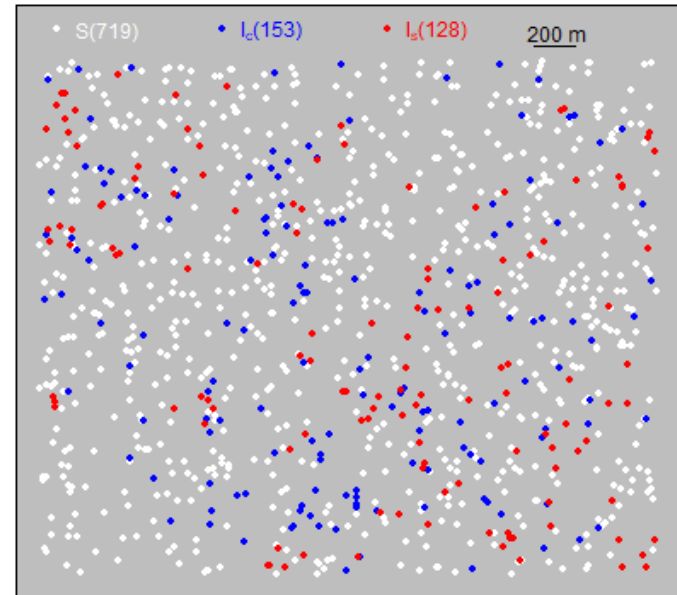
- Here we take  $m = 1000$  draws of  $(\theta_i, Q_i)$  using these as a test-bed of ‘pre-epidemics’ on which to compare controls.

# Results – snapshots of system state

t = 130 (days)

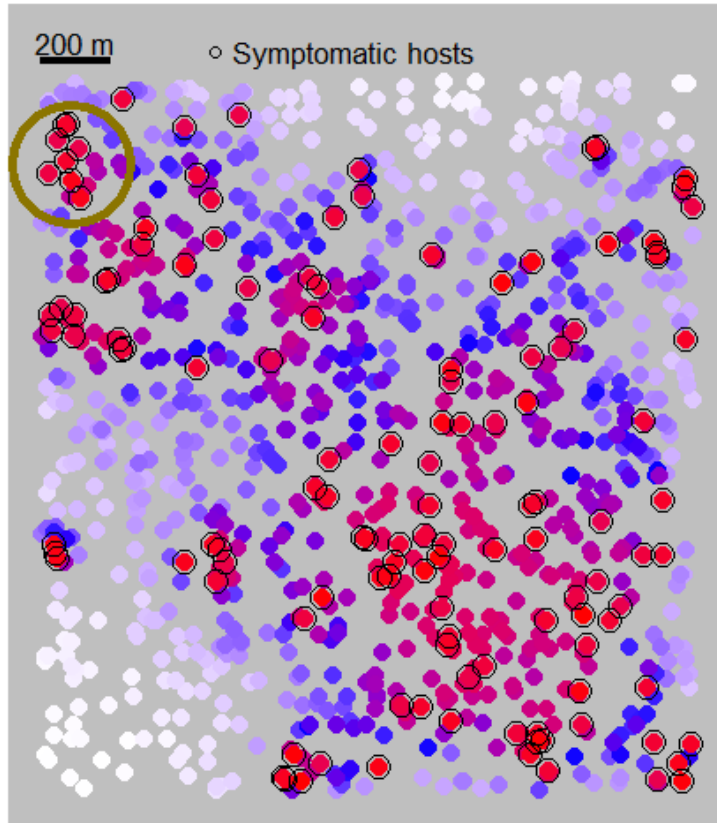


t = 460 (days)

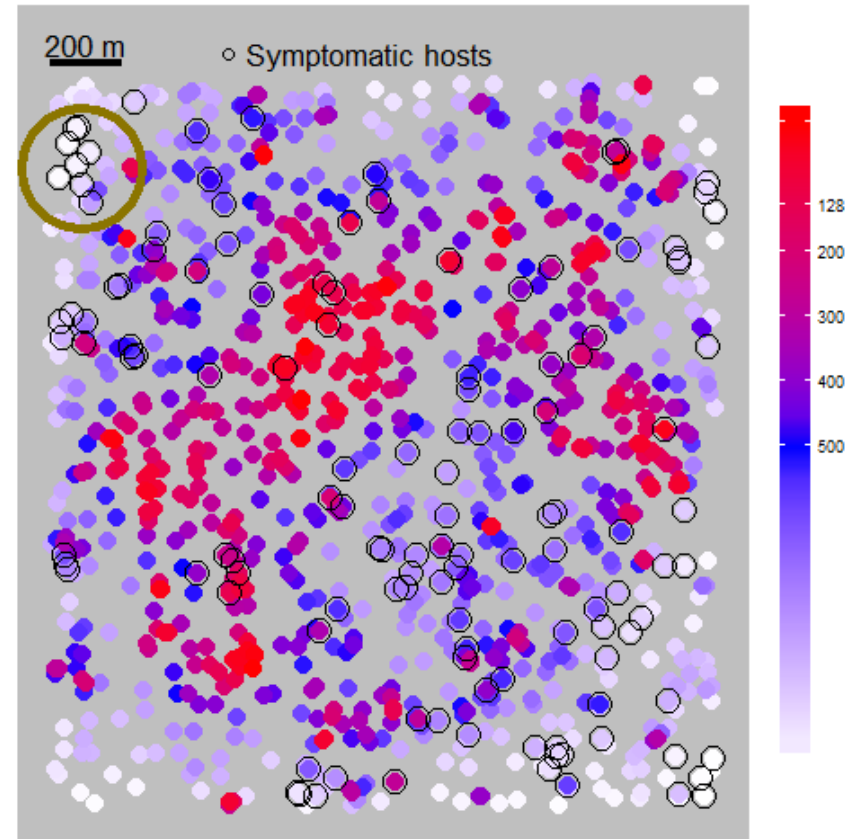


# Maps of risk, hazard threat

## risk

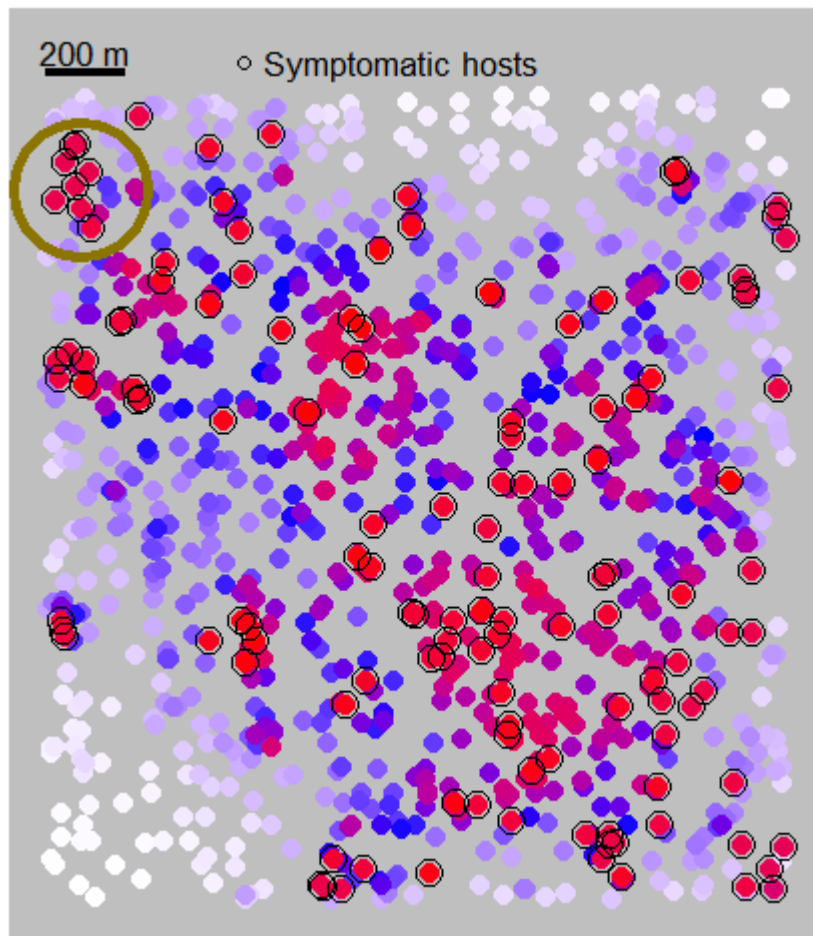


## hazard



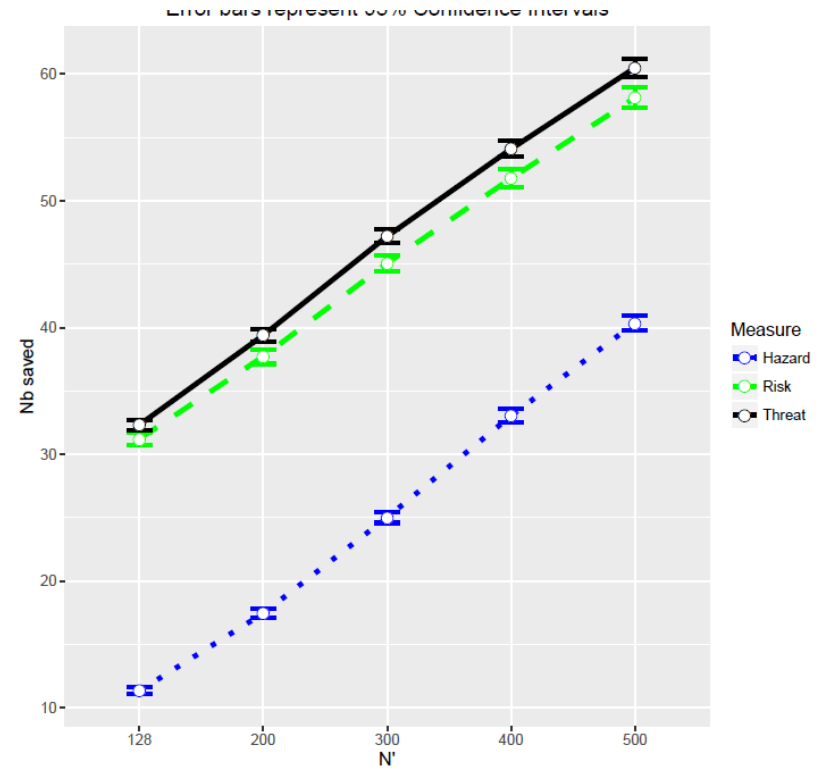
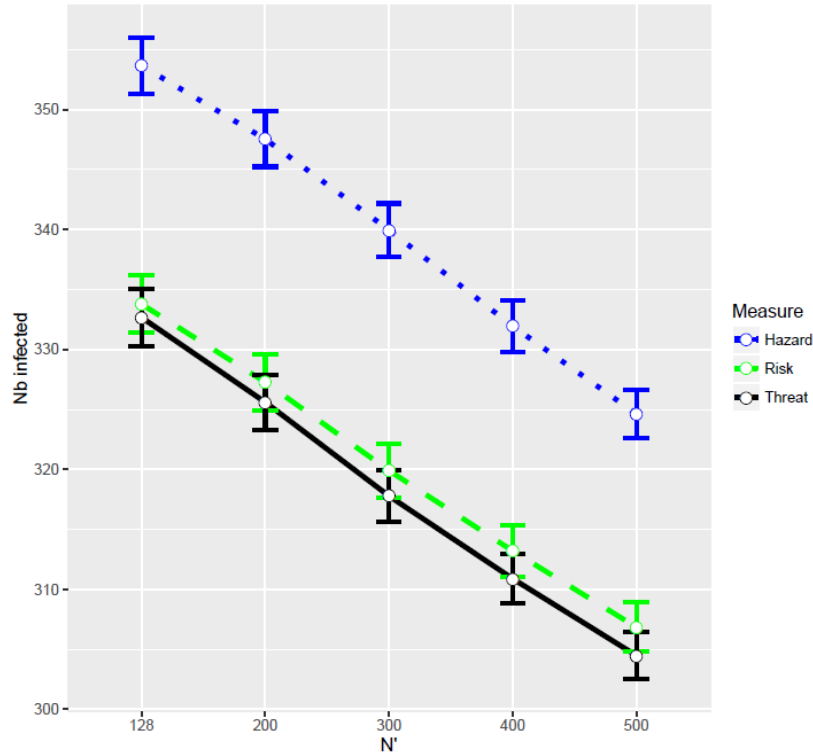
- The hazard values are greatest in regions of low infection while the risk measure is greatest for symptomatic individuals.

# threat

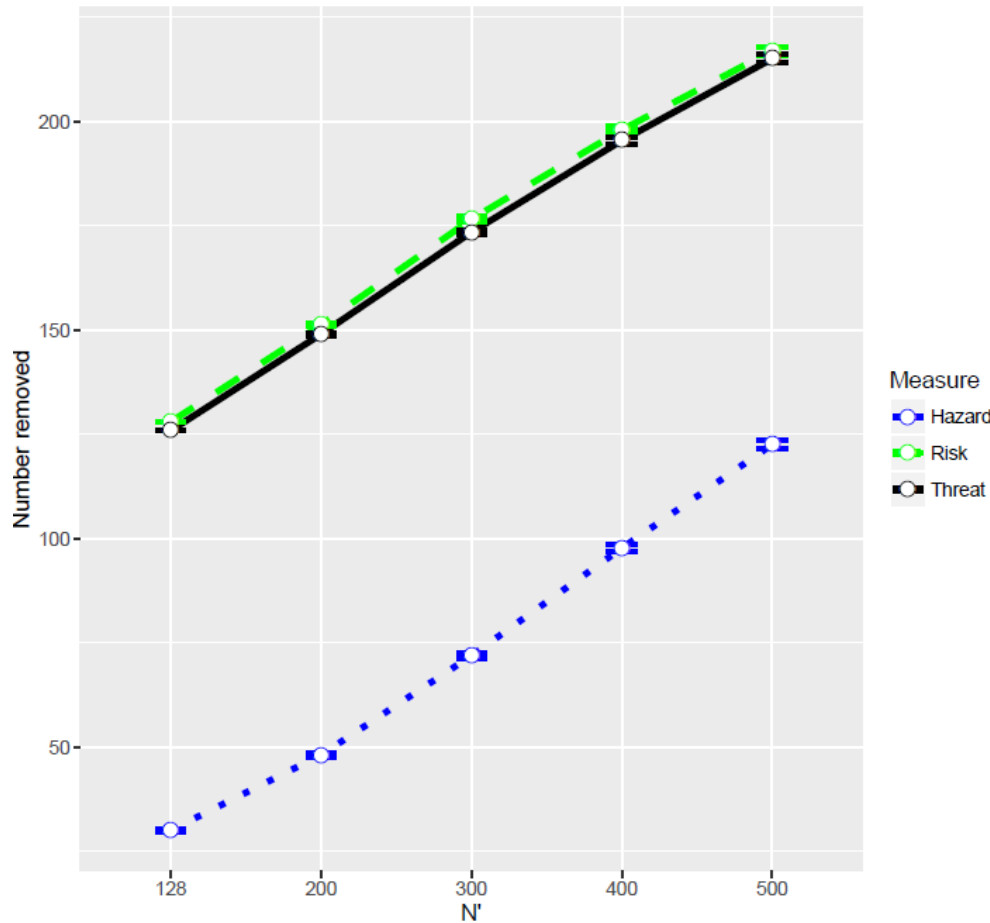


- The dependence of the threat measure on the positions of likely susceptible individuals in relation to an infected host can be discerned
- The infected hosts (circled) in the top left corner of the population naturally exhibit high values of the risk.
- The corresponding threat measure is comparatively lower for these hosts, as a high proportion of their immediate neighbours are already infected.

# $EER(t_A)$ and expected infections



# Expected number of removals

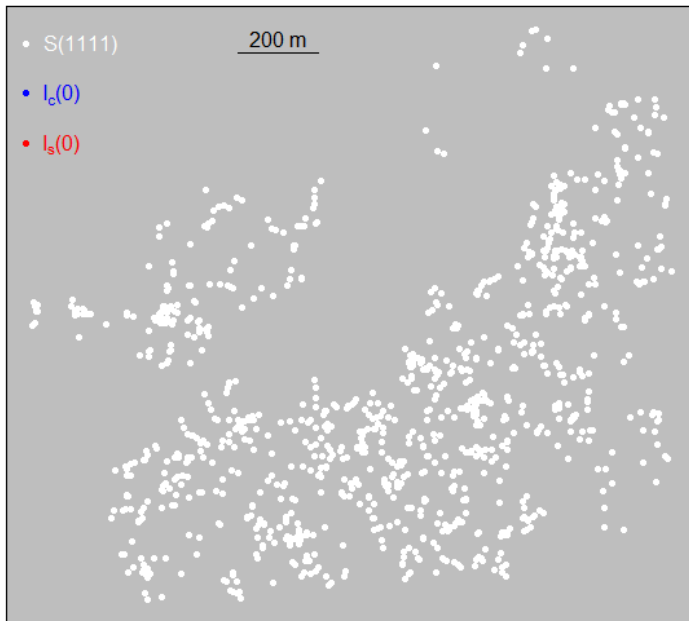


- Marginally improved control provided by threat map at expense of marginally fewer removals.
- Risk and threat generally comparable – suggesting risk of infection is main determinant in threat map.
- Intuitive given uniform distribution of hosts (?) and relatively homogenous appearance of the epidemic.



# Clustered host populations

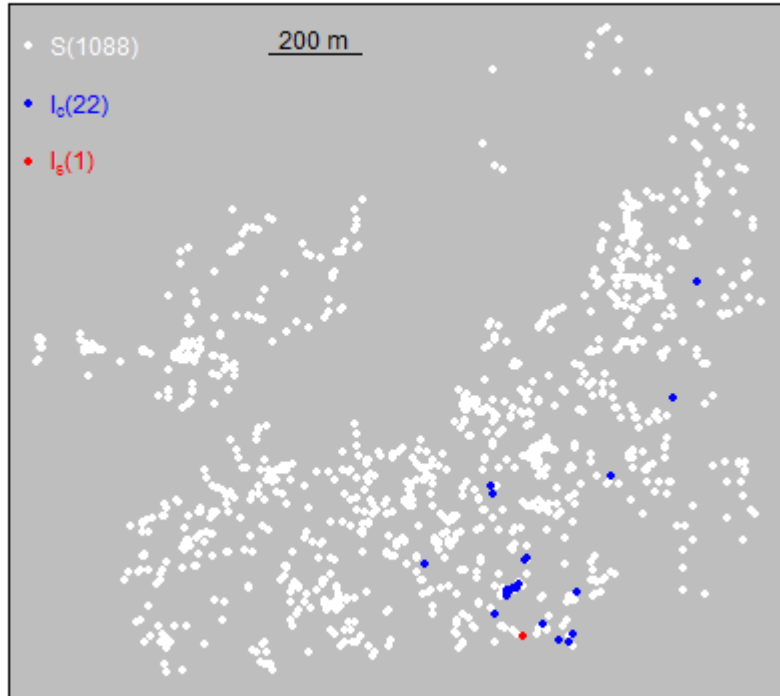
t = 0 (days)



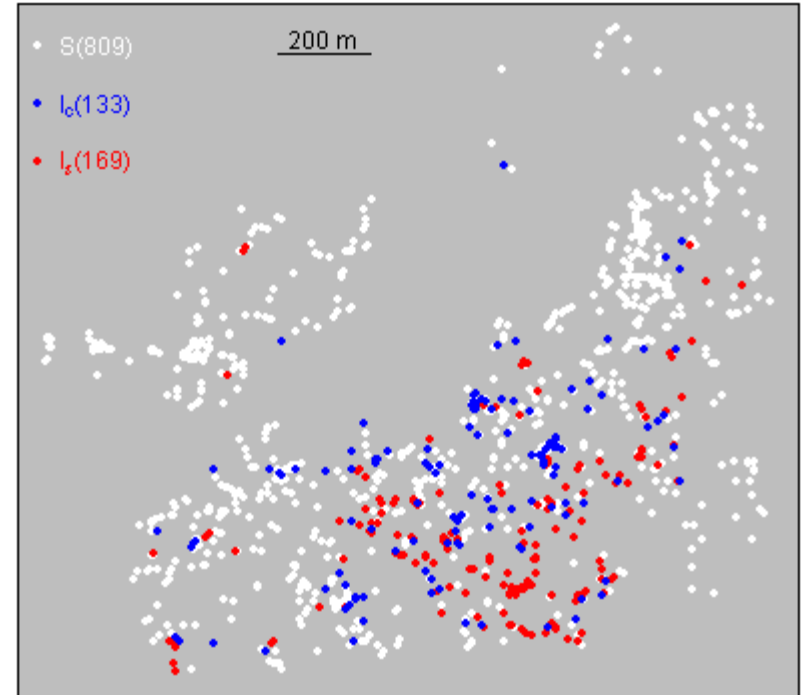
- Citrus locations from Broward county
- 1111 trees spatially distributed
- Citrus canker epidemic on this population analysed by Neri et al (2014)
- Canker typically controlled using ring-culling strategies (not yet considered in this framework but amenable to it)
- Simulate epidemics of 2 types:
  - exponential kernel with primary
  - exponential kernel no primary

# Snapshots with primary

t = 130 (days)

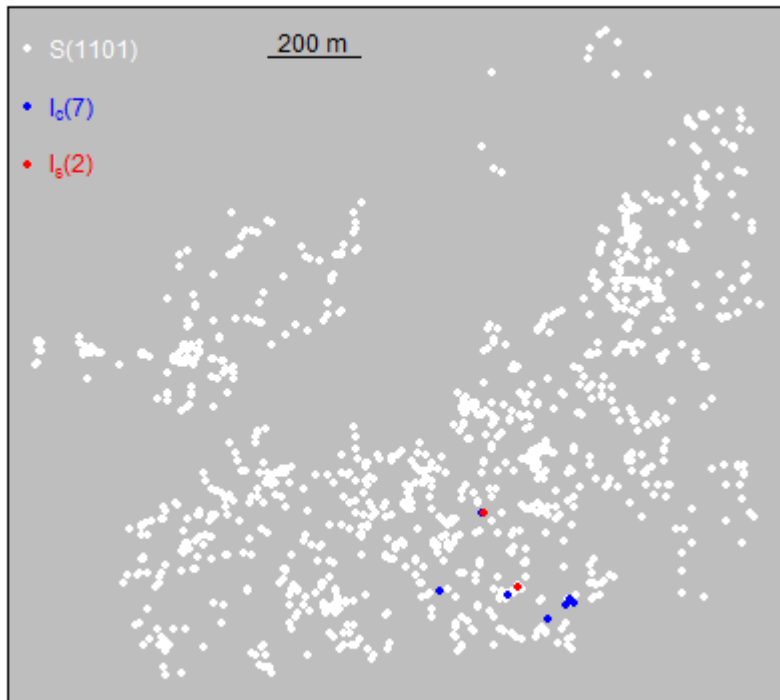


t = 460 (days)

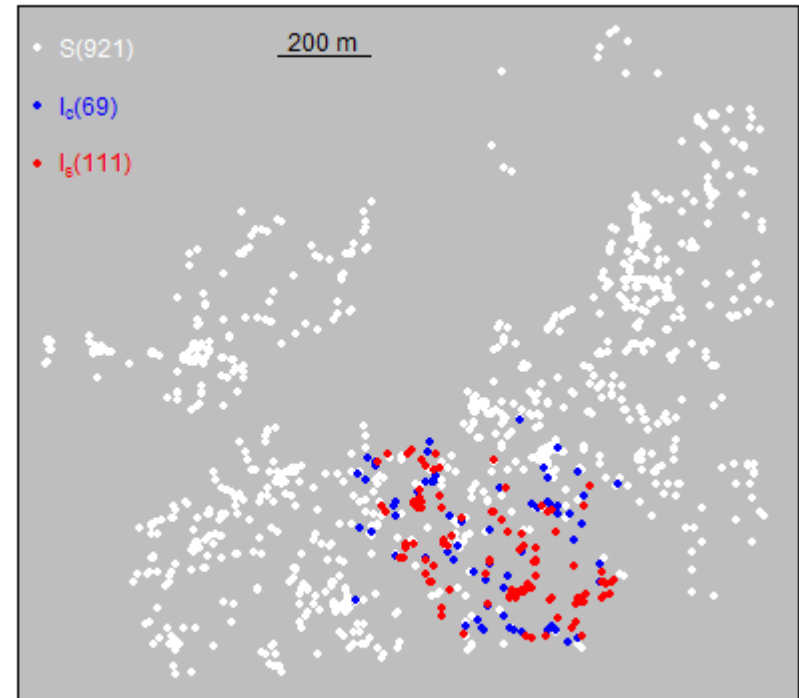


# Snapshots without primary

t = 130 (days)

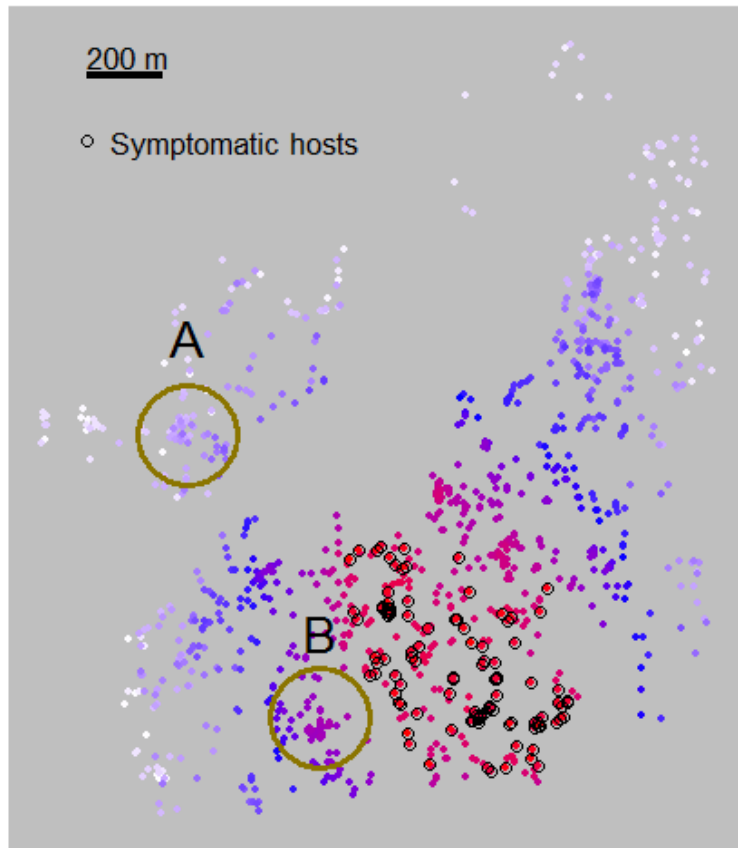


t = 460 (days)

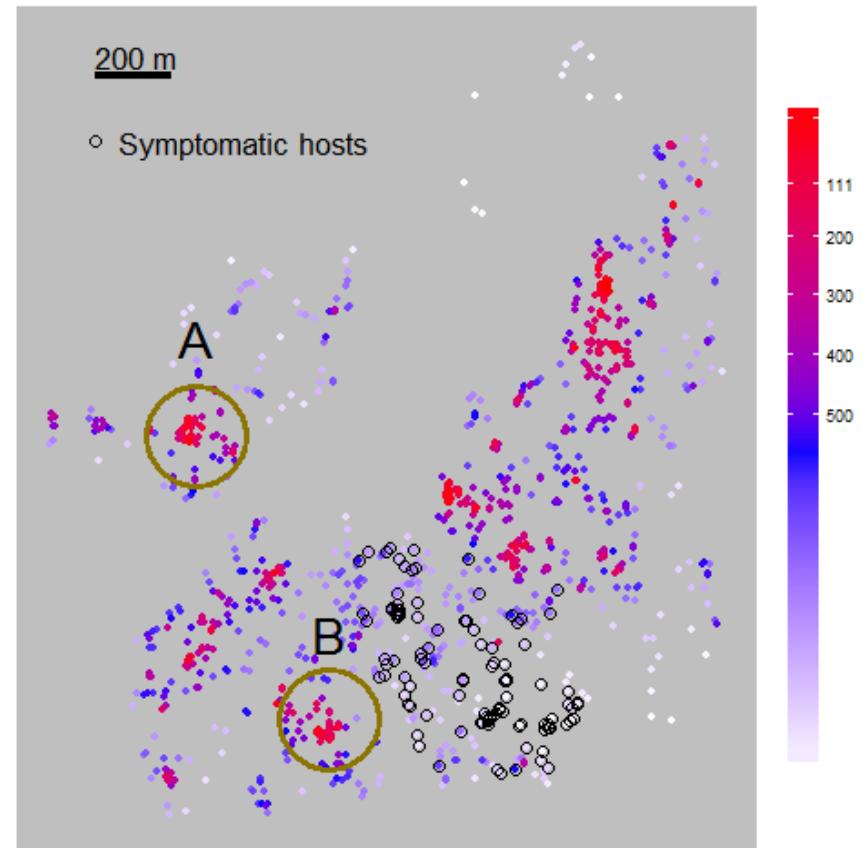


# Maps, no primary

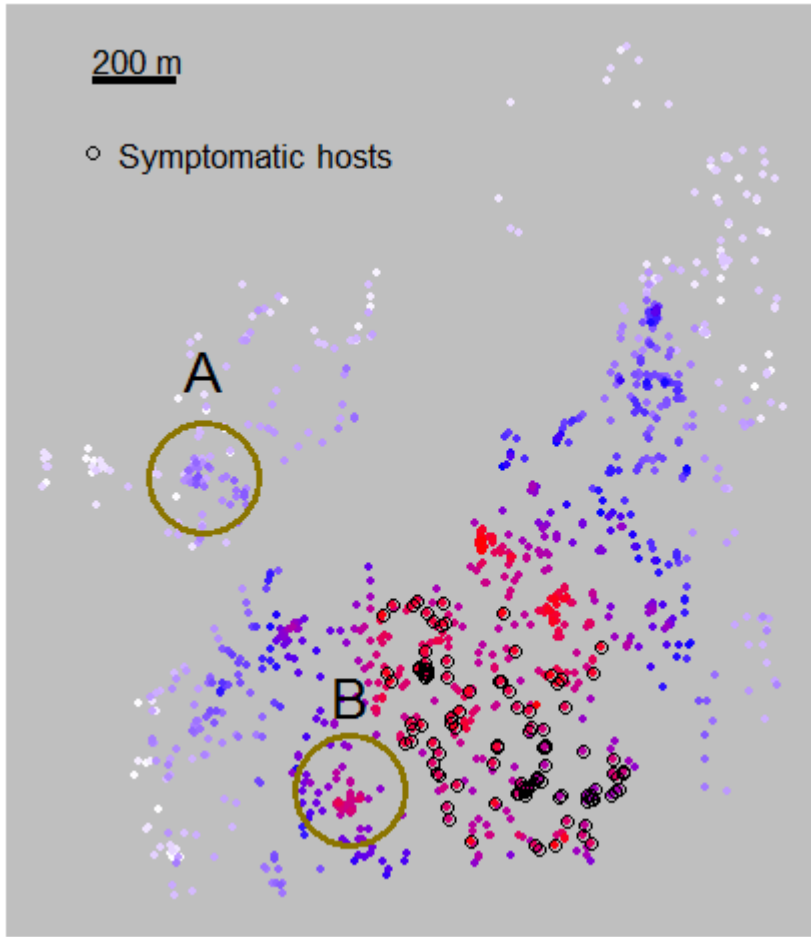
## Risk



## Hazard



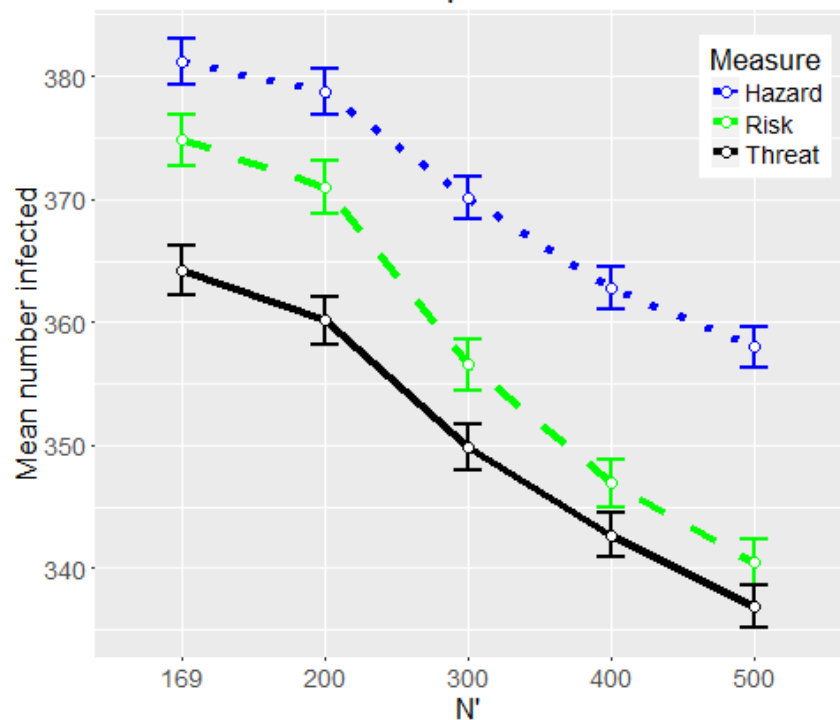
# Threat



- The 111 symptomatic hosts detected during the survey are indicated by the black circles.
- A cluster with intermediate risk (B) leads to high threat due to the high hazard.
- while one with very low risk (A) ends up with relatively low threat even though the hazard is high.

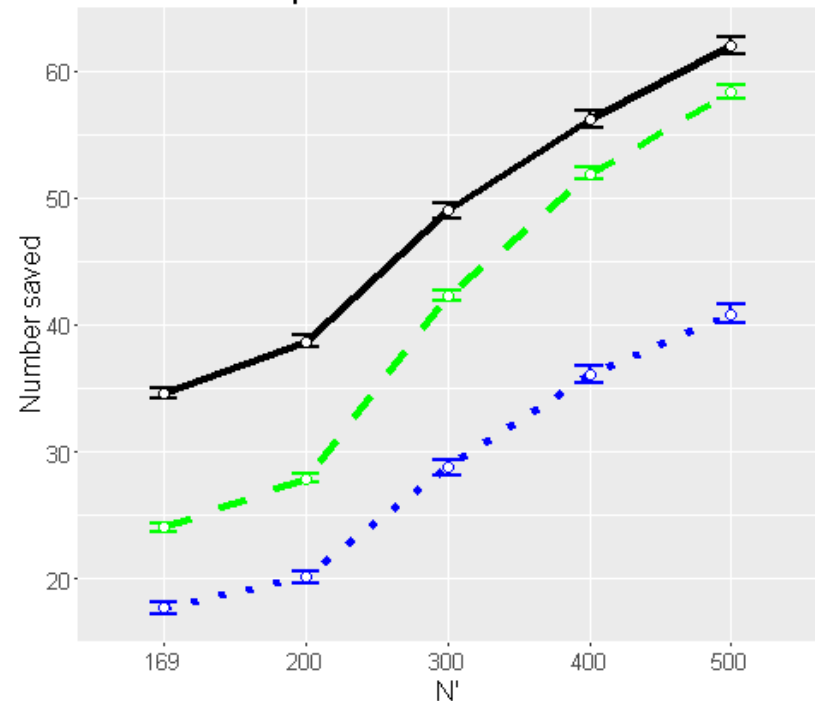
# $EER(t_A)$ and expected infections, Case (I)

Mean number infected depending on the size of resources and prioritisation measure. Error bars represent 95% CI

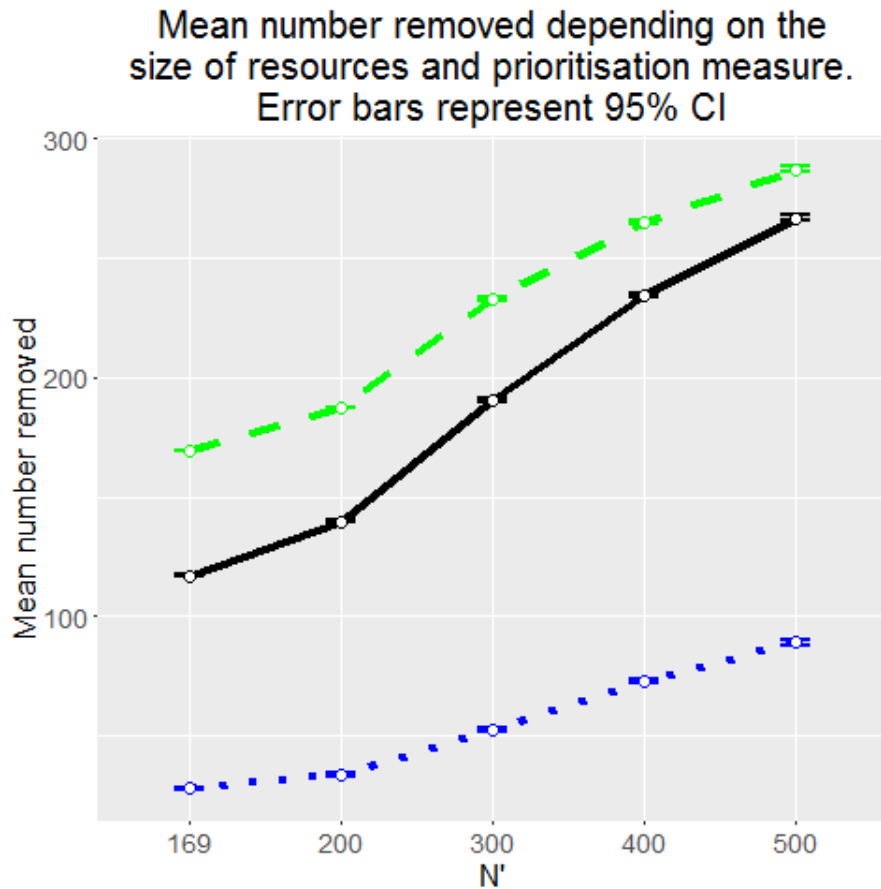


*d*

Mean number removed depending on the size of resources and prioritisation measure. Error bars represent 95% Confidence Intervals



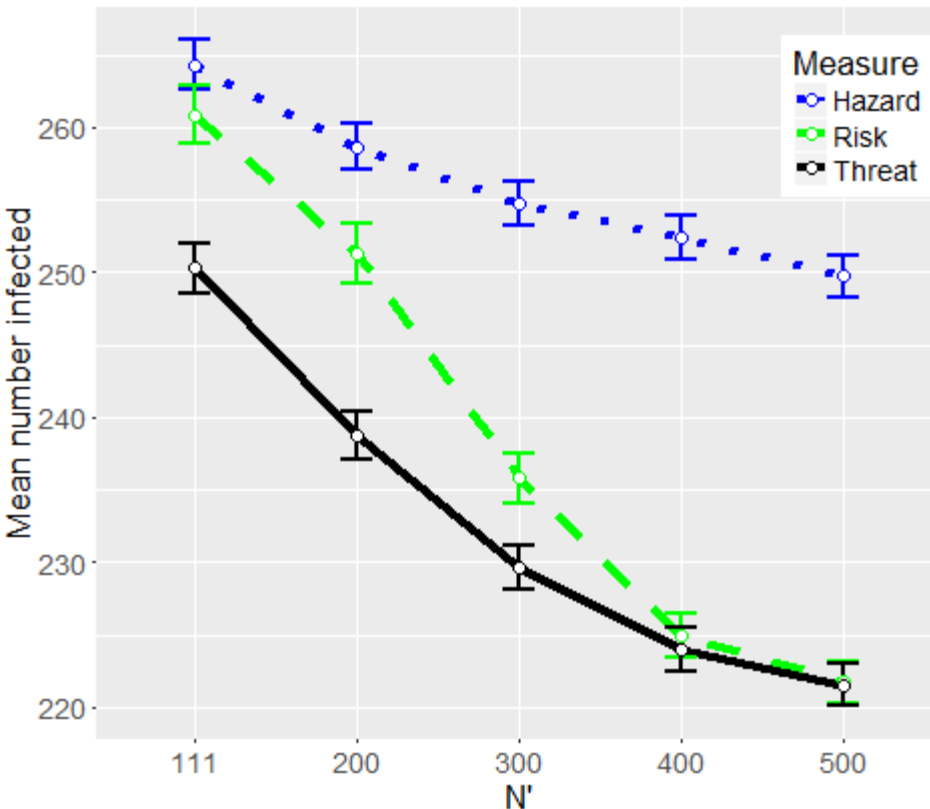
# Expected number of removals, Case (I)



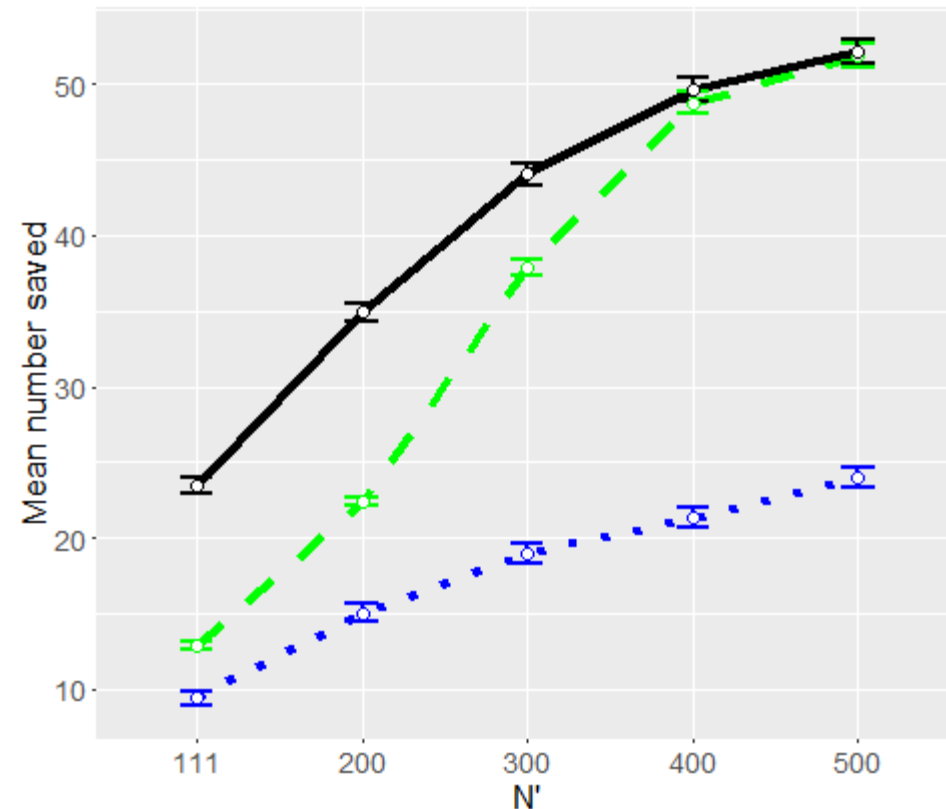
- Difference in performance between the risk and threat measure than was observed for the uniformly distributed population. prioritisation based
- Prioritisation based on the threat map is the most cost-effective control strategy in reducing the impact of the epidemics.
- With scarce resources (lower values of  $N'$ ) the difference between results for the threat and risk measure decreasing as  $N'$  increases.
- The change in the discrepancy between threat and risk maps with increasing  $N'$  is most pronounced in Case (II),

# $EER(t_A)$ and expected infections, Case (II)

Mean number infected depending on the size of resources and prioritisation measure. Error bars represent 95% CI



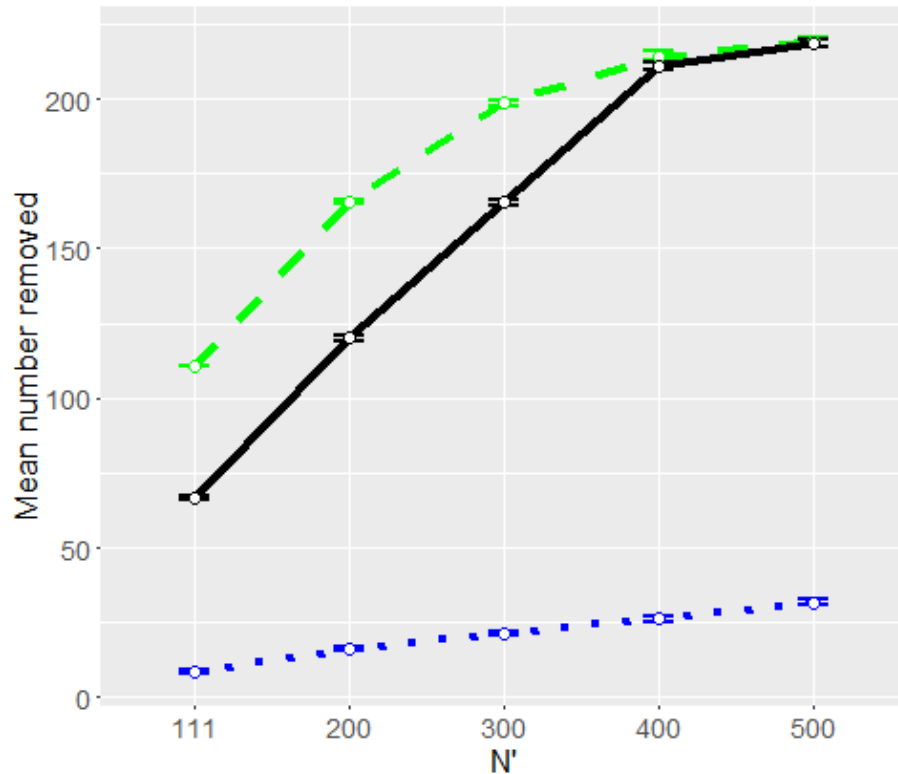
Mean number saved depending on the size of resources and prioritisation measure. Error bars represent 95% CI





# Expected number of removals, Case (II)

Mean number removed depending on the size of resources and prioritisation measure.  
Error bars represent 95% CI



- For small values of  $N'$  the risk map's performance improves little on that of the hazard map but converges to that of the threat map as  $N'$  approaches its maximal value.
- Less removal with the threat compared to the risk and hazard with higher host saved.

# Summing up

- Data augmentation valuable for designing control strategies.
- Removal of hosts based on the threat map is the most effective strategy to reduce the impact of an epidemic – even though fewer hosts are targeted for removal.
- Latent processes (Sellke thresholds) can be used to couple epidemics and subsequently reduce the variability in the difference of control strategies.
- Sample size needed for the estimation is reduced compared to an independent sampling.
- The approach is paralisable .



Thank you

